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# Design Optimization of semi rigid steel framed Structures to AISC- LRFD using Harmony search algorithm

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***واللجنة إذ تمنحه هذه الدرجة فإنها توصيه بتقوى الله ولزوم طاعته وأن يسخر علمه في خدمة دينه ووطنه.***

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# ABSTRACT

The aim of this research is to develop a computer design model which obtains the optimum design of multistorey steel frames by selecting from a standard set of steel sections. Strength constraints of American Institute Steel Construction (AISC)-Load and Resistance Factor Design (LRFD) specification, displacement constraints and size constraint for beam-columns were imposed on frames.

Harmony search (HS) is a recently developed metaheuristic search algorithm that was conceptualized using the musical process of searching for a perfect state of harmony. The harmony in music is analogous to the optimization solution vector, and the musician's improvisations are analogous to local and global search schemes in optimization techniques. The HS algorithm does not require initial values for the decision variables. Furthermore, instead of a gradient search, the HS algorithm uses a stochastic random search that is based on the harmony memory considering rate and the pitch adjusting rate so that derivative information is unnecessary.

The HS algorithm accounts for the effect of the flexibility of the connections and the geometric non-linearity of the members. The semi-rigid connections are modelled with the Frye–Morris polynomial model. Moreover, two steel frames with extended end plate without column stiffeners are designed using HS algorithm. Full Catalog Section (FCS) and Selected Catalog Section (SCS) are used to compare the obtained results.

The results prove that harmony search algorithm is a powerful and effective tools, in comparison with genetic algorithm. Also the comparisons showed that the harmony search algorithm yielded lighter frame in case of rigid and semi-rigid connections for the presented models. In addition, using the Selected Catalog Sections the optimum frames are lighter than that of the Full Catalog Sections. Moreover, HS converges to optimum designs before the maximum numbers of iterations executed in almost designs.

# الملخص

تهدف الرسالة لتطوير نموذج رياضي لإيجاد الحل الأمثل لتصميم المنشآت المعدنية متعددة الطوابق بإستخدام الوصلات المرنة. كما تم استخدام جميع معايير التصميم مثل قيود الإزاحة والترخيم والأبعاد والتحمل حسب المواصفات القياسية للهيئة الأمريكية للمنشآت المعدنية.

ظهرت طريقة البحث عبر التناغم حديثاً من الموسيقي الطبيعية أو الموسيقي الإصطناعية لإيجاد أفضل نغمة موسيقية. حيث أن هذة الطريقة لاتحتاج إلي قيم أولية للبدء في إيجاد الحل الأمثل, وعليه يتم إختيار الحلول بطريقة عشوائية تحت قيود معينة.

طريقة البحث عبر التناغم تأخذ تأثير مرونة الوصلات وتأثير شكل المنحني الغير خطي للعناصر الإنشائية. وتم إستخدام الوصلات الممتدة والغير مدعمة من الأعمدة, وتم نمذجتها بإستخدام متعدد الحدود فراي موريس. كما يتم التصميم بإستخدام كتالوجين أحدهما يشمل جميع العناصر والآخر يفصل العناصر الإنشائية الأحزمة عن الأعمدة وسوف يتم مقارنة الحل الأمثل لكل منهما.

أثبتث طريقة البحث عبر التناغم مدي قوتها بالمقارنة مع الخوارزمية الجنية, لانها تعطي أوزان أقل للمنشآ في حالة الوصلات المثبثة كلياً والوصلات المرنة. بالإضافة إلي ذلك فإن التصميم بإستخدام كتالوج العناصر الإنشائية المختارة يعطي أوزان أقل من كتالوج العناصر المكتملة. وعلاوة علي ذلك فإن طريقة البحث تصل إلي الحل الأمثل قبل الإنتهاء من جميع الدورات المقررة لها.

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| | |
|---|---|
| AISC | : American Institute of Steel Construction. |
| LRFD | : Load and Resistance Factor Design. |
| $M$ | : Bending Moment. |
| $\theta r$ | : Rotation. |
| HS | : Harmony search. |
| PR | : Partial restrain. |
| FR | : Fully restrain. |
| $c_1$ | : Curve fitting constants. |
| $c_2$ | : Curve fitting constants. |
| $c_3$ | : Curve fitting constants. |
| $K$ | : Standardization constant. |
| $M_0$ | : Starting value of connection moment |
| $R_{Kf}$ | : Strain hardening stiffness |
| $a$ | : Scaling factor |
| $C_j$ | : Curve fitting constant |
| GA | : Genetic algorithm. |
| SA | : Simulated annealing. |
| ACO | : Ant colony optimization. |
| *HMS* | : Harmony memory size. |
| *HMCR* | : Harmony memory consideration rate. |
| *PAR* | : Pitch adjusting rate. |
| *HM* | : Harmony memory. |
| $\varphi(x^n)$ | : Unconstraint objective function. |
| rn | : Random number. |
| $X_{SL}$ | : Section list. |
| $t_p$ | : Plate thickness. |
| $d_g$ | : Distances between vertical bolts. |
| d | : Section height. |

$d_b$ : Bolts diameter.

$S_i$ : Secant stiffness.

$\Delta M$ : The incremental moment value.

$\Delta\theta$ : The incremental rotation value.

$W(x)$ : Minimum weight.

$N_g$ : Total numbers of groups.

$Ak$ : Cross-sectional area of member.

$mk$ : Total numbers of members in group.

$\rho_i$ : Density of steel member.

$L_i$ : Length of member.

$\varepsilon$ : Penalty function exponent.

$C_i^t$ : Constraint violations for top-storey displacement.

$C_i^d$ : Constraint violations for interstorey displacement.

$C_i^{sc}$ : Constraint violations for size constraints column to column.

$C_i^{sb}$ : Constraint violations for size constraints Beam to column.

$C_i^{db}$ : Constraint violations for deflection.

$C_i^I$ : Interaction formulas of the LRFD specification.

$N_{jt}$ : Number of joints in the top storey.

$N_s$ : Number of storey's except the top storey.

$N_c$ : Number of beam columns.

$N_{cl}$ : The total number of columns in the frame except the ones at the bottom floor.

$N_f$ : Number of storey.

$\lambda_i^t$ : Displacement constraints at the top storey.

$\lambda_i^d$ : Displacement constraints at the interstorey.

$\lambda_i^{db}$ : Deflection control for each beam.

$\lambda_i^{sc}$ : Size constraint column to column.

$\lambda_i^{sb}$ : Size constraint beam to column.

$d_t$ : Maximum displacement in the top storey.

$d_t^u$ : Allowable top storey displacement.

$d_i$ : Interstorey displacement in storey.

$d_t^u$ : Allowable interstorey displacement.

$d_{db}$ : Maximum deflection for each beam.

$d_{du}$ : Allowable floor girder deflection.

$\sigma_n$ : Upper storey sway.

$\sigma_{n-1}$ : Lower storey sway.

L : Span length.

$d_{un}$ : Depths of steel sections selected for upper floor.

$d_{bn}$ : Depths of steel sections selected for lower floor.

$d_{bf}$ : Beam flange width.

$d_{bc}$ : Column flange width.

$P_u$ : Axial strength.

$\phi_t$ : Resistance factor for tension.

$\phi_c$ : Resistance factor for compression.

$P_n$ : Nominal axial strength.

$\lambda_i^I$ : Strength constraints for beam-column.

$M_{UX}$ : Requires flexural strengths about the major axis.

$M_{Uy}$ : Requires flexural strengths about the minor axis.

$\varphi_b$ : Flexural resistance factor.

$M_{nx}$ : Nominal flexural strength about the major axis.

$M_{ny}$ : Nominal flexural strength about the minor axis.

LTB : Lateral- torisional buckling.

$L_b$ : Unbraced length.

$L_p$ : Unbraced length at the plastic moment.

$L_r$ : Unbraced length at the buckling moment.

$M_p$ : Plastic moment.

$F_y$ : Yield stress of steel.

$Z_x$ : Plastic section modulus.

$C_b$ : Moment coefficient.

$M_r$ : Buckling moment at Lr.

$F_r$ : Compressive residual stress in flange: 10 ksi.

$S_x$ : Elastic section modulus about major axis.

$r_y$ : Governing radius of gyration about minor axis.

$E$ : Modulus of elasticity of steel.

$G$ : Shear modulus of elasticity of steel.

$J$ : Torsional constant.

$A$ : Cross sectional area.

$Cw$ : Warping constant.

$Iy$ : Moment of inertia about Y- axis.

$A_g$ : Cross-sectional area.

$F_{cr}$ : Critical compressive stress.

$\lambda_c$ : Column slenderness parameter.

$G_A$ : Restraint factor at the end of column (at point A).

$G_B$ : Restraint factor at the end of column (at point B).

$Ic$ : Moment of inertia of a column.

$Lc$ : Unsupported length of a column.

$I_B$ : Moment of inertia of a beam.

$L_B$ : Unsupported length of a beam.

FCS : Full Catalog Section.

SCS : Selected Catalog Section.

PSO : Parallel simulating annealing optimization.

# CHAPTER 1 : INTRODUCTION

## 1.1 General.

The structural response of a steel frame is closely related to the behaviour of its beam to column connections. The realistic modelling of a steel frame, therefore, requires the use of realistic connection modelling if an accurate response of the frame is to be obtained. Steel connections are assumed either perfectly pinned or fully rigid in most design of steel frames. This simplification leads to an incorrect estimation of frame behaviour. In fact, the connections are between the two extreme assumptions and possess some rotational stiffness. Bolted and welded connections rotate at an angle due to applied bending moment. This connection deformation has negative effect on frame stability, as it increases drift of the frame and causes a decrease in effective stiffness of the member which is connected to the joint.

An increase in frame drift will multiply the second order (P-Δ) effects of beam column members and thus will affect the overall stability of the frame. Hence, the non-linear features of beam to column connections have important function in structural steel design. AISC-LRFD specification describes three types of steel constructions: rigid-frame (fully restrained), simple framing (unrestrained) and semi-rigid framing (partially restrained) [1]. This specification requires that the connections of partially restrained construction have a flexibility intermediate in degree between the rigidity and the flexibility, and this type of analyses may need non-elastic (non-linear) deformations of structural steel parts.

Most experiments have shown that the $M$-$\Theta r$ curve is non-linear in the whole domain and for all types of connections [2-5]. As a result, modelling of the nodal connection is vital for the design and accuracy in the frame structure analysis. In the present study, the semi-rigid connections are modeled with the Frye–Morris polynomial model [6, 7].

Apart from connection non-linearity's, the effects of geometrical non-linearity of the beams and columns are also of practical interest. Structural analysis that includes geometrical non-linearity is termed second-order analysis or P-delta (P-Δ) analysis. Geometrical non-linearity's occurred when members bend and the structure sways or deflects laterally under loading. The lateral displacement of the column results in second- order moment to the column which can be calculated from the applied load multiplied by the appropriate lateral displacement. Hence, the non-linear features of beam to column connections have important function in structural steel design.

Harmony search is a music-based metaheuristic optimization algorithm. It was inspired by the observation that the aim of music is to search for a perfect state of harmony, such as during jazz improvisation. Harmony Search (HS) algorithm is applied to obtain the optimum design of steel frames. The design algorithm obtains the minimum weight of the frame by selecting a standard set of steel sections such as American Institute of Steel Construction (AISC). Strength constraints of AISC-Load and Resistance Factor Design (LRFD) specification, displacement constraints and also size constraint for beam-columns will be imposed on frames [1, 8 and 9].

## 1.2 Problem statement.

The processes of obtaining the optimum design of structures are very complex to solve by hand, due to large number of design variables, objectives. Typically, the design is limited by constraints such as the choice of material, feasible strength, displacements, deflection, size constraints, load cases, support conditions, and true behaviour of beam to column connection. Hence, one must decide which parameters can be modified during the optimization process. Usually, structural optimization problems involve searching for the minimum of the structural weight in steel structure. This minimum weight design is subjected to various constraints with respect to performance measures, such as stresses and displacements, and restricted by practical minimum cross-sectional areas or dimensions of the structural members or components.

This Research considers a Harmony Search (HS) algorithm based approach for optimizing the size and configuration of structural systems with discrete design variables [10-13].

## 1.3 Motivation

Design optimization methods have been used to obtain more economical designs since 1970s [14-15]. Numerous algorithms have been developed for accomplishing the optimization problems in the last four decades. The early works on the topic mostly use mathematical programming techniques or optimality criteria with continuous design variables. These methods utilize gradient of functions to search the design space.

Today's competitive world has forced the engineers to realize more economical designs and designers to search/develop more effective optimization techniques. As a result, heuristic search methods emerged in the first half of 1990s [16-17].

A new meta-heuristic search algorithm called harmony search has been developed by Geem et al. [10]. Harmony search (HS) bases on the analogy between the performance process of natural music and searching for solutions to optimization problems. HS can be easily programmed and adopted for engineering problems.

**The main Advantages of HS are summarized as:**

1. HS obtains a new design considering all existing designs.
2. HS takes into account each design variable independently.
3. HS does not code the parameters, HS uses real value scheme

4. HS updates its memory after each design is generated.

## 1.4 The objectives of this research.

The main aim of the current study to develop a computer design model which obtains the optimum frame weight by selecting a standard set of steel sections and satisfy strength constraints of AISC Load and Resistance Factor Design (LRFD) specification, displacement constraints, beam deflection and also size constraint for columns and beam-column were imposed on frames. Harmony search method will be used in this research to obtain the optimum design.

**The objectives of this research are:**

1. Develop a computer model which designs steel framed structures with rigid and semi rigid connections.
2. Build up the Harmony search algorithm and connect it to the design model.
3. Carry out validation and verification of the developed models.
4. Compare the optimization results with conventional optimization technique.

## 1.5 Research scope.

**The scope of study for this research includes:**

1. Linear and geometric non-linear behaviour of steel Structures.
2. Two-dimensional planer frame.
3. Semi-rigid beam-column connection.
4. Rigid column base.

## 1.6 Methodology

**To achieve the objectives of this research, the following tasks will be executed:**

1. Conduct a literature survey for optimization using harmony search technique, modes of simulating semi rigid connection.
2. Build up the computer model.
3. Build up the suitable harmony search algorithm.
4. Conduct the verification and validation the developed models.
5. Compare the optimization results with conventional design using structural design packages.
6. Draw conclusion and recommendation.

## 1.7 Content of thesis.

Chapter 2 of this thesis discusses the literature review. Chapter 3 discusses Harmony Search algorithm. Chapter 4 describes modelling of steel frame structure. Chapter 5 describes formulation of the optimization problem. Chapter 6 presents Analysis result and discussion. In the end, Chapter 7 presents conclusions and future direction.

# CHAPTER 2 : LITERATURE REVIEW

## 2.1 Introduction.

The structural response of a steel frame is closely related to the behaviour of its beam-to-column connections. Therefore, the realistic modelling of a steel frame requires the use of realistic connection modelling if an accurate response of the frame is to be obtained. Experiments have shown that the actual behaviour lies somewhere between fully fixed and fully pinned [2-5]. The effect of connection flexibility must be taken into account in the analysis and design procedures. A beam-to-column connection is generally subject to axial force, shear force, bending moment and torsion. However, for practical purposes, only the effect of moment on the rotational deformation of connections needs to be considered. This is because the effect of torsion can be excluded in-plane study. Moreover, the effect of axial and shear forces are usually small compared to that of the bending moment [6].

## 2.2 AISC-LRFD specification of connections.

AISC-LRFD specification describes three types of steel constructions: simple framing (unrestrained), rigid-frame (fully restrained-FR) and semi-rigid framing (partially restrained-PR) [1].

### 2.2.1 Simple connections.

A simple connection transmits a negligible moment across the connection. In the analysis of the structure, simple connections may be assumed to allow unrestrained relative rotation between the framing elements being connected. A simple connection shall have sufficient rotation capacity to accommodate the required rotation determined by the analysis of the structure. Inelastic rotation of the connection is permitted.

### 2.2.2 Moment connections.

A moment connection transmits moment across the connection. Two types of moment connections are specified below.

1. Fully-Restrained (FR) Moment Connections transfers moment with a negligible rotation between the connected members. In the analysis of the structure, the connection may be assumed to allow no relative rotation. An FR connection shall have sufficient strength and stiffness to maintain the angle between the connected members at the strength limit states.

2. Partially-Restrained (PR) Moment Connections transfer moments, but the rotation between connected members is not negligible. In the analysis of the structure, the force-deformation response characteristics of the connection shall be included. The response characteristics of a PR connection shall be

documented in the technical literature or established by analytical or experimental means. The component elements of a PR connection shall have sufficient strength, stiffness, and deformation capacity at the strength limit states.

Figure 2.1 shows that the connection rotates by an amount $\Theta_r$ when a moment $M$ is applied. The angle $\Theta_r$ corresponds to the relative rotation between the beam and the column at the connection. The rotational distortion of the connection affects the drift of the frame and brings about redistribution of moments between column and beam. As a result, it is certainly more realistic to assume semi-rigid connection models for beam-to column connections in the analysis and design of steel frames.

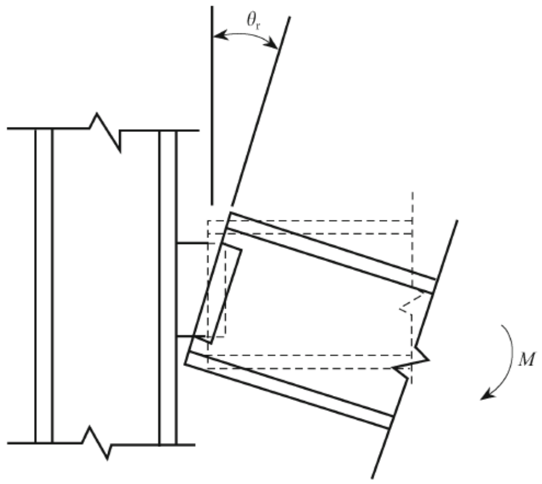


**Figure 2. 1: Rotational deformation of connection.**

## 2.3 Types of beam-column connections.

There are several types of beam to column connections, which are commonly used in fabrication steel work; namely single web angle, double web angle, header plate, top and seat angles, top and seat angle with double web angles, extended end pate without column stiffeners, extended end plate with column stiffeners and T-stub connection.

### 2.3.1 Single web angle connection.

This connection is made by an angles connected to the beam web and then connected to the column flange, as shown in Figure 2.2. This connection represents a very flexible joint [7].

Major geometric parameters, which influence single web angle behaviour, have been identified as:

I. Number of beam web bolts.

II. Angle plate thickness and depth.

III. Column flange or web thickness.

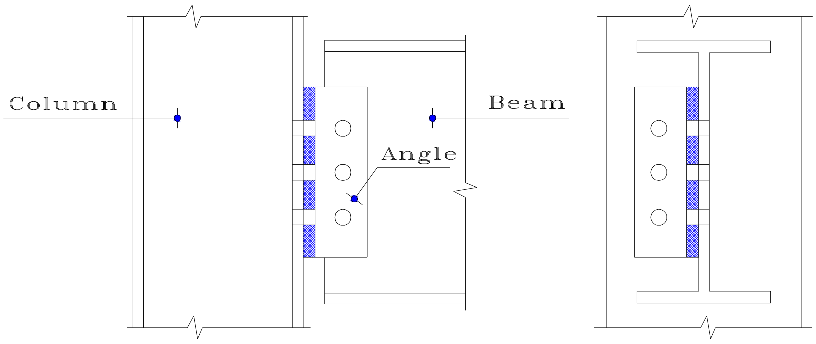


**Figure 2. 2: Single web angle connection.**

### 2.3.2 Double web angle connection.

This connection is made by two angles connected to the beam web and then connected to the column flange, as shown in Figure 2.3. The earliest tests on double web-angle connections were conducted by Rathbun [18], using rivets as fasteners. Nowadays, high strength bolts are used [19].

Major geometric parameters, which influence double web angle behaviour, have been identified as:

I. Number of beam web bolts.

II. Angle thickness and depth.

III. Column flange or web thickness.

IV. Gauge distance of column bolts.

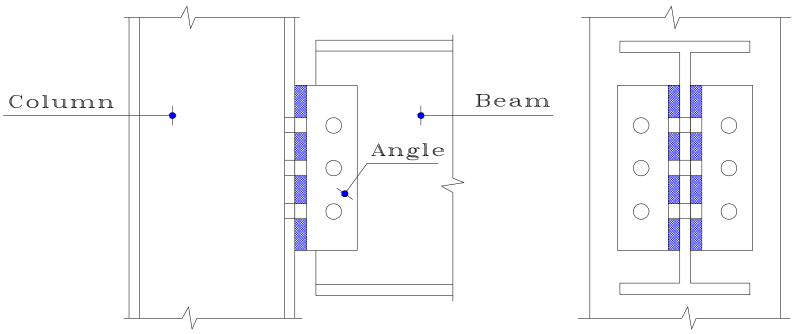


**Figure 2. 3: Double web angle connection.**

### 2.3.3 Header plate connection.

A header plate connection consists of an end plate, whose length is less than the depth of the beam, welded to the beam and bolted to the column; also, it may be welded after coping the beam web, as shown in Figure 2.4. A header plate connection used to transfer the reaction of the beam to the column. The behaviours of these connections are similar to those of double web angle connections [2].

Major geometric parameters, which influence header plate behaviour, have been identified as:

I. Plate thickness.

II. Plate depth.

III. Beam-web thickness.

IV. Gauge distance of column bolts.

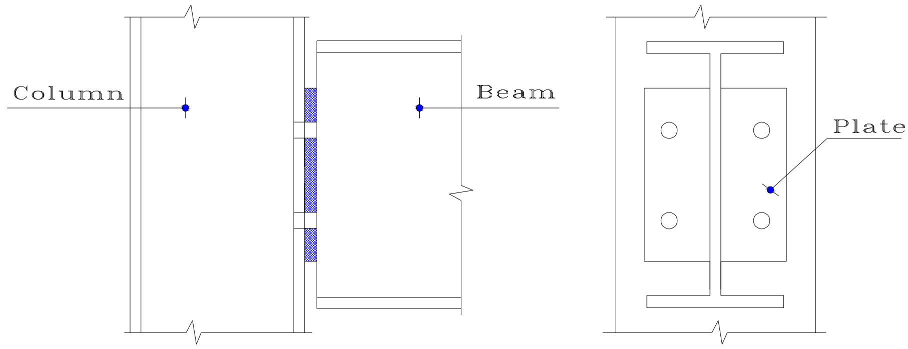


**Figure 2. 4: Header Plate connection.**

### 2.3.4 Top and seat angle connection.

The AISC specification describes the top and seat angle connections as (a) the seat angle transfers only vertical reaction and should not give significant restraining moment at the end of the beam; (b) the top angle is merely used for lateral stability and is not considered to carry any gravity loads. A typical top and seat angle connection is shown in Figure 2.5.

Major geometric parameters, which influence top and seat angle behaviour, have been identified as:

I. Number of beam flange bolts.

II. Thickness of angle

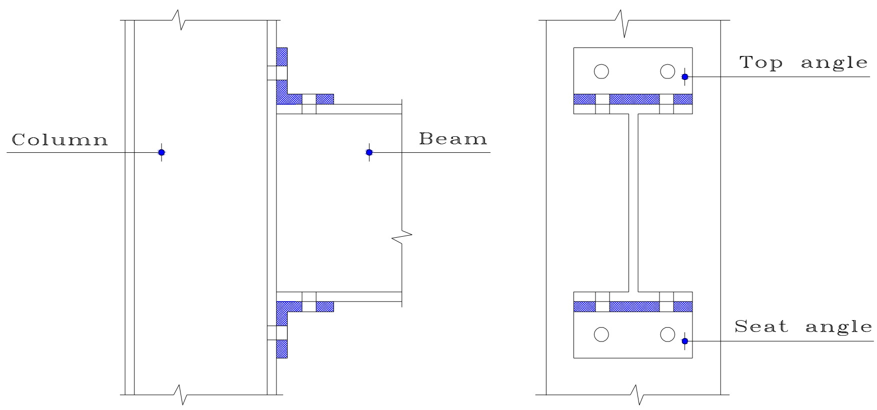


**Figure 2. 5: Top and seat angle connection.**

### 2.3.5 Top and seat angle with double web angle connection.

Top and seat angle can be coupled with double web angles to take heavier loads as shown in Figure 2.6.

Major geometric parameters, which influence top and seat angle with double web angle behaviour, have been identified as:

I. Thickness and depth of angles.

II. Column flange or web thickness.

III. Gauge distance of bolts in vertical angle leg.

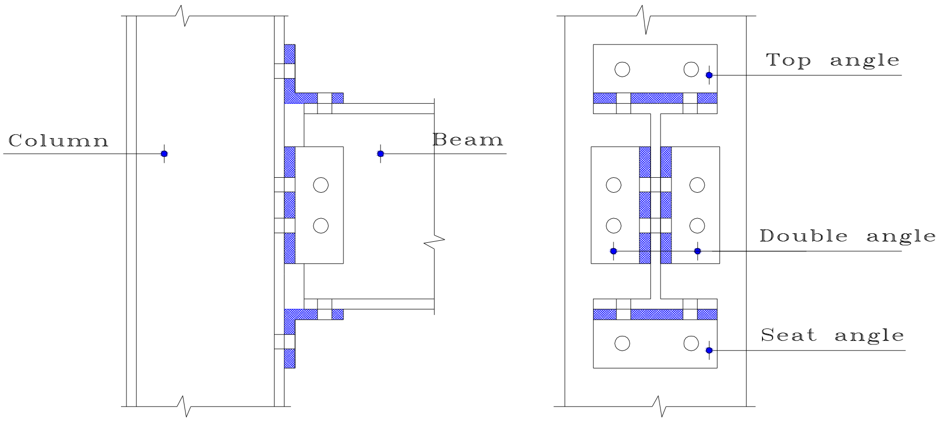


**Figure 2. 6: Top and seat angle with double web angle connection.**

### 2.3.6 Extended end plate without column stiffeners connection.

The extended end plate connections are welded to the beam end along both flanges and web in the fabricator's shop and bolted to the column in the field. This type of connection is extending in both tension and compression sides, as shown in Figure 2.7.

Major geometric parameters, which influence extended end plate without column stiffeners behaviour, have been identified as:

I. Plate thickness.

II. Column flange thickness.

III. Moment arm for column flange bolts.

### 2.3.7 Extended end plate with column stiffeners connection.

The extended end plate connections are welded to the beam end along both flanges and web in the fabricator's shop and bolted to the column in the field and stiffened column flange. This type of connection is extending in both tension and compression sides, as shown in Figure 2.8.

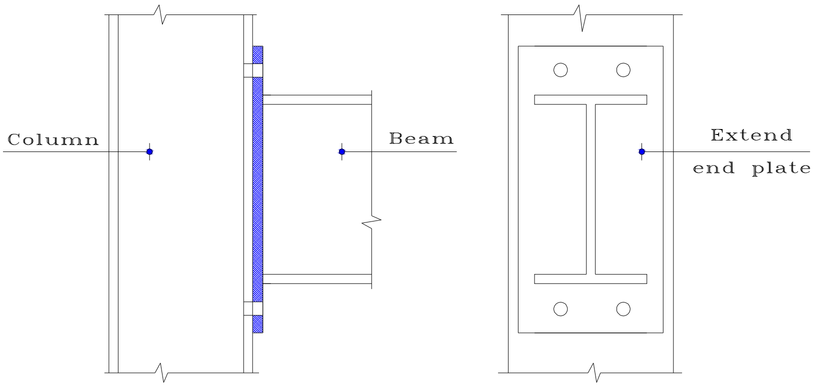


**Figure 2. 7: Extended end plate without column stiffeners connection.**

Major geometric parameters, which influence extended end plate with column stiffeners behaviour, have been identified as:

I. Plate thickness.

II. Column flange thickness.

III. Moment arm for column flange bolts.

IV. Column stiffness depth and thickness.

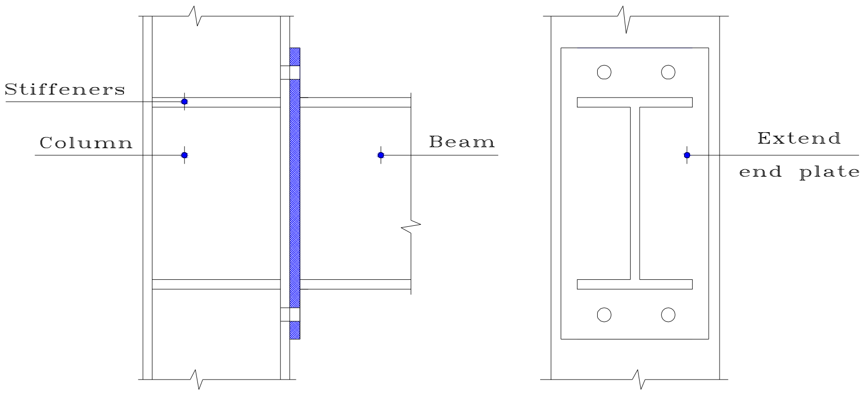


**Figure 2. 8: Extended end plate with column stiffeners connection.**

### 2.3.8 T-stub connection.

T-stub connection are similar to top and seat angles connection configuration expect that the cut of Tee section is employed instead of angles as shown in Figure 2.9. This connection represents a very rigid joint [7].

Major geometric parameters, which influence T-stub behaviour, have been identified as:

I. T-stub thickness.

II. Width of T-stub.

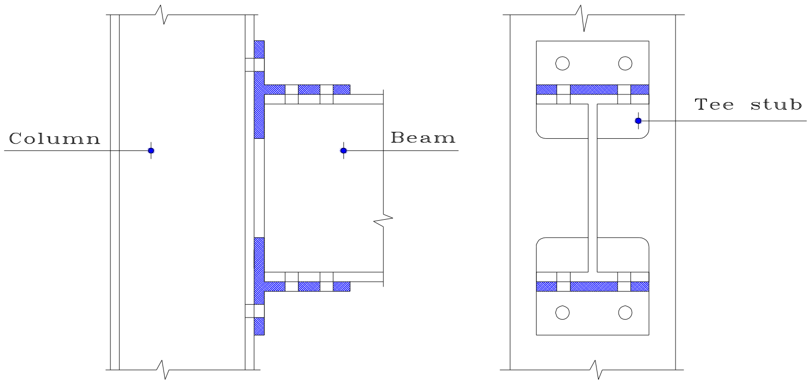


**Figure 2. 9: T-stub connection.**

## 2.4 Behaviour of steel connections.

All types of connections exhibit non-linear moment-rotation behaviour that falls between the two extreme cases of fully fixed and ideally pinned. Experiments have shown the relationship between the moment and the beam-to-column joint rotation is non-linear in nature [2-5]. In general, the connection is dependent on the geometric parameters of the elements used in the connections, such as bolt size and dimensions of end plate or angle sections etc... . Relative moment-rotation curves of extensively used semi-rigid connections are shown in Figure. 2.10 [7].

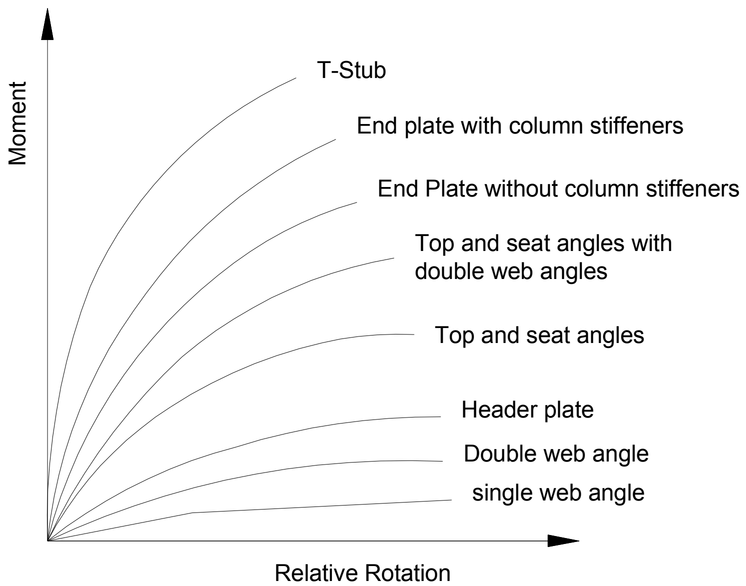


**Figure 2. 10: Moment-rotation curves of semi-rigid connections.**

## 2.5 Mathematical modelling of semi-rigid connections.

There are several mathematical connection models as the following:

### 2.5.1 Linear model.

The linear models were proposed by Batho, Rathbun, and Baker [18]. The bi-linear models were proposed by Melchers, Kaur, Romstad, Subrmanian, Lui and Chen [20]. The piecewise linear models were proposed by Razzaq.

### 2.5.2 Polynomial model.

Frye and Morris [7] used odd power polynomial to represent the moment-rotation curve as,

$$\theta_r = c_1(KM)^1 + c_2(KM)^3 + c_3(KM)^5 \qquad \text{....................................(2.1)}$$

Where $K$ is standardization constant which depends upon connection type and geometry; $c_1$, $c_2$, $c_3$ are the curve fitting constants.

### 2.5.3 Cubic B-spline model.

This model can be fit test data well. However, a large number of data are required in the curve fitting process [21].

### 2.5.4 Power model.

The power model Proposed by Batho and Lash. has the following expression :

$$\theta_r = aM^b \qquad \text{....................................(2.2)}$$

Where the two parameters a and b are used to fit the curve, subjected to the condition, $a > 0$, $b > 1$

### 2.5.5 Exponential model.

This model gives a good curve fitting with test curve up to and including the strain-hardening range.

Chen and lui multi-paramter model has form:

$$M = \sum_{j=1}^{m} c_j \left[ 1 - \exp\left( -\frac{|\theta_r|}{2ja} \right) \right] + M_0 + R_{kf}|\theta_r| \qquad \text{....................................(2.3)}$$

Where $M_0$ =strating value of connection moment, $R_{Kf}$ = strain hardening stiffness, a = scaling factor, $C_j$= curve fitting constant.

## 2.6 Optimization of steel structure.

Today's competitive world has forced the engineers to realize more economical designs and designers to search/develop more effective optimization techniques. As a result, heuristic search methods emerged in the first half of 1990s. Heuristic search algorithms have been applied to various optimum design problems since then. Genetic algorithms (GAs), simulated annealing (SA) and ant colony optimization (ACO) that appeared as optimization tools are quite effective in obtaining the optimum solution of discrete optimization problems. One of the applications of heuristic search methods is optimum design of steel frames [16, 17 and 22-25].

Several researches focusing on the behaviour of the connections have been made to correlate the data obtained by experimental and theoretical analysis.

Saka, (2009) [13], studied the optimum design of rigid steel frames using harmony search algorithm according to British Standard BS5950. The harmony search method is a numerical optimization technique developed recently that imitates the musical performance process which takes place when a musician searches for a better state of harmony. Jazz improvisation seeks to find musically pleasing harmony similar to the optimum design process, which seeks to find the optimum solution. The optimum design algorithm developed imposes the behavioral and performance constraints in accordance with BS5950. The algorithm presented selects the appropriate sections for beams and columns of the steel frame from the list of 64 Universal Beam sections and 32 Universal Column sections of the British Code. The optimum results obtained by the harmony search algorithm are lighter than the one obtained by the simple genetic algorithm.

Hayalioglu and Degertekin (2005) [26] presented a minimum cost design of steel frames with semi-rigid connections and column bases via genetic optimization. The design algorithm obtains the minimum total cost, which comprises total member, plus connection costs by selecting suitable sections from a standard set of steel sections such as American Institute of Steel Construction (AISC) wide-flange (W) shapes. Displacement and stress constraints of AISC-Load and Resistance Factor Design (LRFD) specification and size constraints for beams and columns are imposed on the frame. The Frye and Morris polynomial model and a linear spring model are used for semi-rigid connections and column bases respectively. It was found from the results that reducing connection stiffness causes increase in both optimum frame cost and the sway. The reason for this is that more flexible connections increase the displacements of the frame, but these displacements are adjusted to their restrictions by the optimization process assigning larger sections to the members.

Lee and Geem (2005) [11] suggested a structural optimization method based on the harmony search (HS) meta-heuristic algorithm, which was conceptualized using the musical process of searching for a perfect state of harmony. The HS algorithm does not require initial values and uses a random search instead of a gradient search, so derivative information is unnecessary. Various truss examples with fixed geometries are presented to demonstrate the effectiveness and robustness of the new method. The results indicated that the suggested technique is a powerful search and optimization method for solving structural engineering problems compared to conventional mathematical methods or genetic algorithm-based approaches.

Kameshki and Saka (2003) [27] proposed optimum design using a genetic algorithm for nonlinear steel frames with semi-rigid connections. A genetic algorithm based optimum design method is presented for nonlinear multistory steel frames with semi-rigid connections. The design algorithm obtains optimum frame by selecting appropriate sections from standard steel section tables while satisfying the serviceability and strength limitations specified in British standard BS5950. The algorithm accounts for the effect of the flexibility of the connections and the geometric non-linearity of the members. The semi-rigid connections are modeled with the Frye–Morris polynomial model.

The result indicates that when the overall gravity loading is much larger compared to lateral loading and is dominant in the design of the frame, linear semi-rigid frames are lighter than linear rigid frames. On the other hand, if the overall gravity loading is not that large compared to lateral loading, geometric nonlinearity in the frame design yields lighter frames compared to linear frames.

## 2.7 Harmony search algorithm in structural engineering.

A new meta-heuristic search algorithm called harmony search has been developed recently. Harmony search (HS) bases on the analogy between the performance process of natural music and searching for solutions to optimization problems. HS was developed by Geem et al. [10] for solving combinatorial optimization problems. HS can be easily programmed and adopted for engineering problems. Although HS has been applied to a diverse range of engineering problems; such as river flood model [28], vehicle routing [29], optimal design of water distribution networks [30], optimal scheduling of multiple dam system [31], minimization for slope stability analysis [32], optimized the truss structures with discrete design variables [33], harmony search algorithm for optimum geometry design of geodesic domes and rigid steel frames [13, 34].

## 2.8 Concluding remarks.

Based on the study, which carried out on the connection behaviour and the connection types from the literature, it is found that extended end plate connections are widely used in steel structures. The literature review showed that Frye-Morris polynomial model is a powerful tool to represent the moment-rotation behaviour of a connection. On the other hand, a new meta-heuristic algorithm harmony search HS showed powerful results in structure optimization problem.

# CHAPTER 3 : HARMONY SEARCH ALGORITHM

## 3.1 Introduction.

Over the last four decades, a large number of algorithms have been developed to solve various engineering optimization problems. Most of these algorithms are based on numerical linear and nonlinear programming methods that require substantial gradient information and usually seek to improve the solution in the neighborhood of a starting point. These numerical optimization algorithms provide a useful strategy to obtain the global optimum in simple and ideal models. Many real-world engineering optimization problems, however, are very complex in nature and quite difficult to solve using these algorithms. If there is more than one local optimum in the problem, the result may depend on the selection of an initial point, and the obtained optimal solution may not necessarily be the global optimum. Furthermore, the gradient search may become difficult and unstable when the objective function and constraints have multiple or sharp peaks. The computational drawbacks of existing numerical methods have forced researchers to rely on meta-heuristic algorithms based on simulations to solve engineering optimization problems. The common factor in meta-heuristic algorithms is that they combine rules and randomness to imitate natural phenomena. To solve difficult and complicated real-world optimization problems, however, new heuristic and more powerful algorithms based on analogies with natural or artificial phenomena must be explored.

The following sections, discuss a brief overview of some existing meta-heuristic algorithms. Then the harmony search will be explained in details.

## 3.2 Heuristic optimization techniques.

Broadly speaking, all heuristic search algorithms are inspired from natural phenomenon. The name of each heuristic method is indicative of its underlying principle.

### 3.2.1 Genetic algorithm (GA).

Genetic algorithms (GA) are based on evolution theory of Darwin's. They were proposed by Holland [22]. The main principle of GAs is the survival of robust ones and the elimination of the others in a population. GAs are able to deal with discrete optimum design problems and do not need derivatives of functions, unlike classical optimization. However, the procedure for the genetic algorithm is time consuming and the optimum solutions may not be global ones, but they are feasible both mathematically and practically. They were used for the optimum design of semi-rigid steel frames under the actual constraints of design codes [26, 27 and 35-37].

### 3.2.2 Simulating annealing algorithm (SA).

Simulating annealing (SA) is an accepted local search optimization method. Local search is an emerging paradigm for combinatorial search which has recently been shown to be very effective for a large number of combinatorial problems. It is based on the idea of navigating the search space by iteratively stepping from one solution to one of its neighbours, which are obtained by applying a simple local change to it. The SA algorithm is inspired by the analogy between the annealing of solids and searching the solutions to optimization problems. SA was developed by Metropolis et al. [23] and proposed by Kirkpatrick et al. [24] for optimization problems. SA was applied to the optimum design of steel frames under the actual design constraints and loads of code specifications [38-43].

### 3.2.3 Ant colony optimization algorithm (ACO).

Ant colony optimization (ACO) is an application of ant behaviour to the computational algorithms and is able to solve discrete optimum structural problems. It also has additional artificial characteristics such as memory, visibility and discrete time. ACO was originally put forward by Dorigo et al. [25] for optimization problems. The applications of ACO to the structural optimization were about the optimal design of planar/space trusses and frames [44-46].

### 3.2.4 Harmony search optimization algorithm.

Recently, Geem et al. [10] developed a new harmony search (HS) meta-heuristic algorithm that was conceptualized using the musical process of searching for a perfect state of harmony. The harmony in music is analogous to the optimization solution vector, and the musician's improvisations are analogous to local and global search schemes in optimization techniques. The HS algorithm does not require initial values for the decision variables. Furthermore, instead of a gradient search, the HS algorithm uses a stochastic random search that is based on the harmony memory considering rate and the pitch adjusting rate (defined in harmony search meta-heuristic algorithm section) so that derivative information is unnecessary. Compared to earlier meta-heuristic optimization algorithms, the HS algorithm imposes fewer mathematical requirements and can be easily adopted for various types of engineering optimization problems. The following sections present the basics of harmony search algorithm.

## 3.3 Basic of harmony search algorithm.

Harmony is defined as an attractive sound made by two or more notes being played at the same time. Do, Re, Mi, Fa, Sol, La, and Si are called notes which represent a single sound. The HS algorithm imitates musical improvisation process where the musicians try to find a better harmony. All musicians always desire to attain the best harmony, which could be accomplished by numerous practices. The pitches of the instruments are changed after the each practice.

In music improvisation, each player sounds any pitch within the possible range, together making one harmony vector as shown in Figure. 3.1. If all the pitches make a good harmony, that experience is stored in each player's memory, and the possibility to make a good harmony is increased next time. Similarly, in engineering optimization, each decision variable initially chooses any value within the possible range, together making one solution vector. If all the values of decision variables make a good solution, that experience is stored in each variables memory, and the possibility to make a good solution is increased next time.

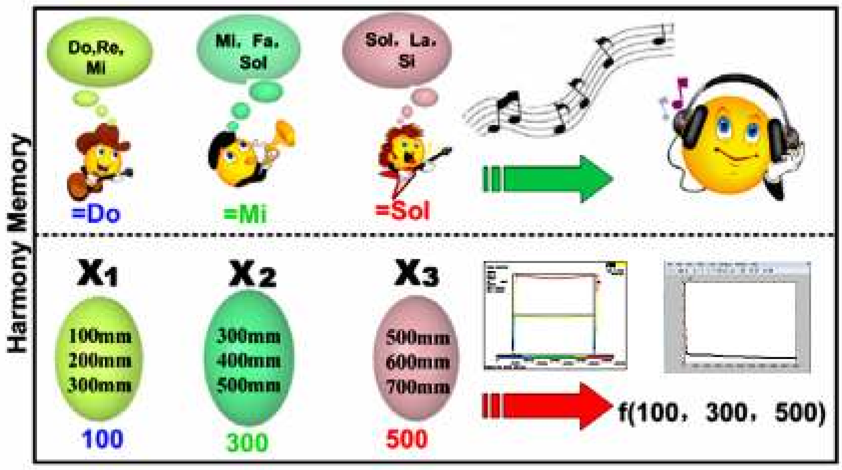


**Figure 3. 1: Analogy between music improvisation and engineering optimization.**

## 3.4 Harmony search optimization algorithm in steel structures.

Figure 3.2 illustrates the analogy between music improvisation and steel design. As explained by Lee and Geem [11], harmony memory (*HM*) is the most important part of HS. Jazz improvisation is the best example for clarifying the harmony memory. Many jazz trios consist of a guitarist, double bassist and pianist. Each musician in the trio has different pitches: guitarist [Fa, Mi, La, Sol, Do]; double bassist [Mi, Do, La, Si, Re]; pianist [Si, Re, Mi, La, Do]. Let guitarist randomly play Sol out of its pitches [Fa, Mi, La, Sol, Do], double bassist Si out of [Mi, Do, La, Si, Re] and pianist Re [Si, Re, Mi, La, Do]. Therefore, the new harmony [Sol, Si, Re] becomes another harmony (musically G-chord).

If the new harmony is better than the existing worst harmony in the *HM*, new harmony is included in the *HM* and the existing worst harmony is excluded from the *HM*. The process is repeated until the best harmony is obtained.

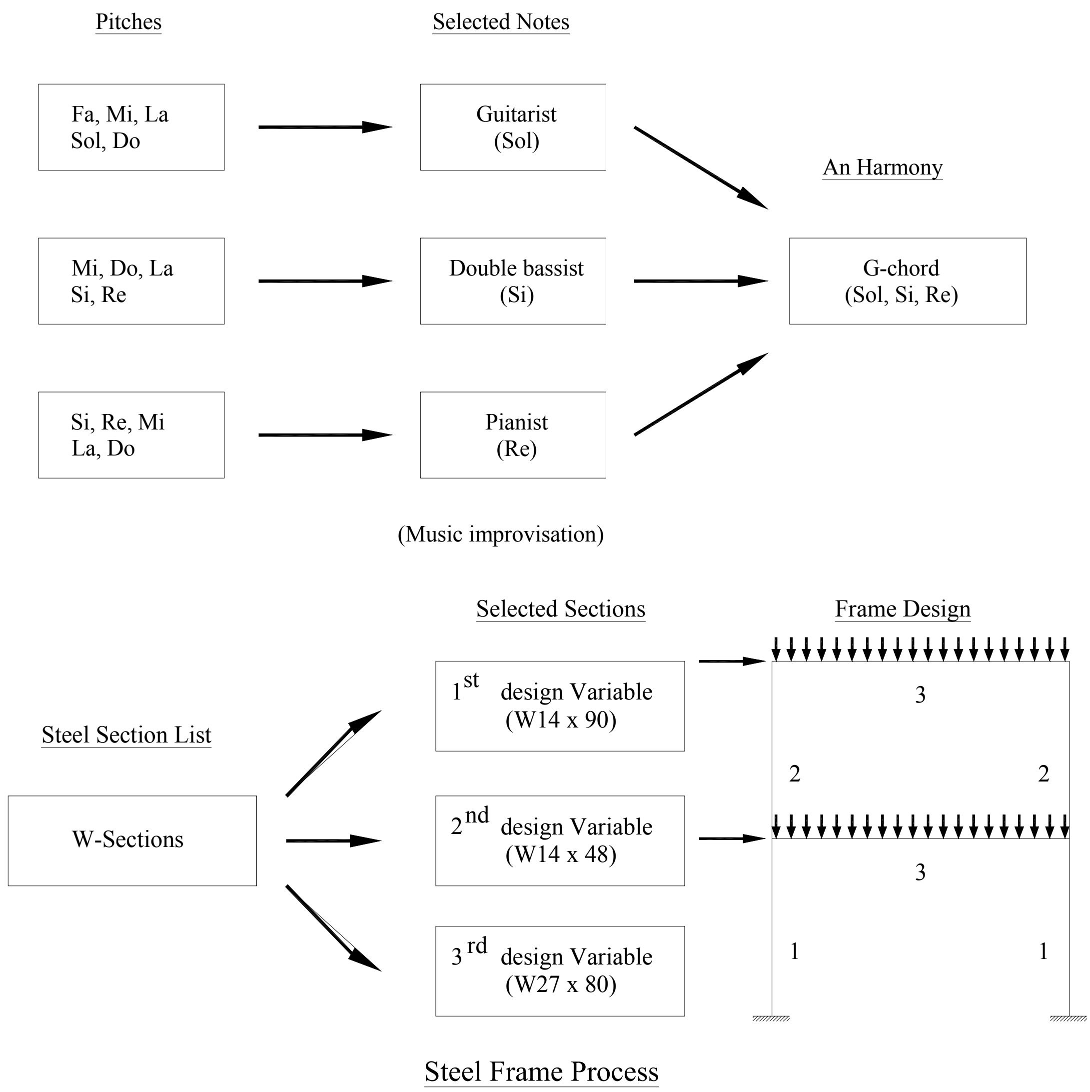


**Figure 3. 2: Analogy between harmony memory and steel frame design process**

For example, in case of a steel frame design process, which consists of three different design variables, the first design variable is the columns of the first storey, the second design variable is the columns of the second storey and the third design variable is the all beams. The design variables are selected from a standard set of steel sections such as American Institute of Steel Construction (AISC) wide-flange (W) shapes. Let us assume W14×90, W14×48 and W27×80 are selected from the section list as the first, second and third design variables respectively. Thus, a new steel design is created [W14×90, W14×48, and W27 × 80]. If the new design is better than existing worst design which is the one with the highest objective function value, the new design is included and worst design is excluded from the steel design process. This procedure is repeated until terminating criterion is satisfied.

An analogy between the music improvisation process and the optimum design of steel frames can be established in the following way: The harmony denotes the design vector while the different harmonies during the improvisation represent the different design vectors throughout the optimum design process. Each musical instrument denotes the design variables (steel sections) of objective function. The pitches of the instruments represent the design variable's values (steel section no.). A better harmony represents local optimum and the best harmony is the global optimum. The following sections describe the harmony search steps.

### 3.4.1 Initialize the harmony search parameters.

The HS algorithm parameters are chosen in this step, they are selected depending on the problem type. The harmony search comprises a number of parameters. These parameters are as follows;

- Harmony memory size (*HMS*).
- Harmony memory consideration rate (*HMCR*).
- Pitch adjusting rate (*PAR*).
- Stopping criteria (number of improvisation).

### 3.4.2 Initialize harmony memory.

The harmony memory (*HM*) matrix is filled with randomly generated designs as the size of the harmony memory size (*HMS*).

Harmony memory matrix is initialized. Each row of harmony memory matrix contains the values of design variables which are randomly selected feasible solutions from the design pool for that particular design variable. Hence, this matrix has n columns where n is the total number of design variables and *HMS* rows which is selected in the first step. *HMS* is similar to the total number of individuals in the population matrix of the genetic algorithm. The harmony memory matrix has the following form:

$$HM = \begin{bmatrix} x_1^1 & x_2^1 & \cdots & x_{ng-1}^1 & x_{ng}^1 \\ x_1^2 & x_2^2 & \cdots & x_{ng-1}^2 & x_{ng}^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_1^{HMS-1} & x_2^{HMS-1} & \cdots & x_{ng-1}^{HMS-1} & x_{ng}^{HMS-1} \\ x_1^{HMS} & x_2^{HMS} & \cdots & x_{ng-1}^{HMS} & x_{ng}^{HMS} \end{bmatrix} \begin{matrix} \rightarrow \varphi(x^1) \\ \rightarrow \varphi(x^2) \\ \rightarrow \vdots \\ \rightarrow \vdots \\ \rightarrow \varphi(x^{HMS-1}) \\ \rightarrow \varphi(x^{HMS}) \end{matrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.1)$$

Each row represents a steel design in the *HM*. $x^1$, $x^2$,…..,$x^{HMS-1}$ ,$x^{HMS}$ and $\varphi(x^1)$, $\varphi(x^2)$,…, $\varphi(x^{HMS-1})$, $\varphi(x^{HMS})$ are designs and the corresponding unconstrained objective function value, respectively. The steel designs in the *HM* are sorted by the unconstrained objective function values which are calculated by using Eqn. (3.1) (i.e. $\varphi(x^1)$, $\varphi(x^2)$,.., $\varphi(x^{HMS})$). The aim of using *HM* is to preserve better designs in the search process.

### 3.4.3 Improvise a new harmony.

A new harmony $\left[x^{nh}\right]=\left\lfloor x_1^{nh}, x_2^{nh}, \ldots\ldots.. x_{ng}^{nh}\right\rfloor$ is improvised from either the *HM* or entire section list. Three rules are applied for the generation of the new harmony. These are *HMCR*, *PAR* and *rn*. In the *HMCR*, the value of first design variable $x_1^{nh}$ for the new harmony is chosen from any value in the *HM* (i.e. $\left\lfloor x_1^1, x_1^2, x_1^{HMS-1}, x_1^{HMS}\right\rfloor$ or entire section list [$x_{SL}$]). [$x_{SL}$] which represents the section list. The other design variables of new harmony $\left\lfloor x_2^{nh}, x_{ng-1}^{nh}, \ldots\ldots.. x_{ng}^{nh}\right\rfloor$ are chosen by the same rationale. *HMCR* is applied as follows

$$\begin{cases} x_i^{nh} \in \left\{x_i^1, x_i^2, \ldots\ldots. x_i^{HMS-1}, \quad x_i^{HMS}\right\} & if\ rn \leq HMCR \\ x_i^{nh} \in x_{sl} & if\ rn > HMCR \end{cases} \qquad \ldots\ldots\ldots\ldots\ldots(3.2)$$

At first, a random number (*rn*) uniformly distributed over the interval [0,1] is generated. If this random number is equal or less than the *HMCR* value, *i-th* design variable of new design [$x^{nh}$] selected from the current values stored in the *i-th* column of *HM*. If *rn* is higher than *HMCR*, *i-th* design variable of new design [$x^{nh}$] is selected from the entire section list [$X_{SL}$]. For example, an *HMCR* of 0.90 shows that the algorithm will choose the *i-th* design variable (i.e. steel section) from the *HM* or from the entire section list with a 10% probability. A value of 1.0 for *HMCR* is not appropriate because of 0% possibility that the new design may be improved by values not stored in the *HM* [11].

Any design variable of the new harmony, $\left[x^{nh}\right]=\left\lfloor x_1^{nh}, x_2^{nh}, \ldots\ldots.. x_{ng}^{nh}\right\rfloor$ which obtained by the memory consideration is examined to determine whether it is pitch-adjusted or not. Pitch adjustment is made by pitch adjustment ratio (*PAR*) which investigates better design in the neighbouring of the current design. *PAR* is applied as follows current stored steel sections in the *i-th* column of the *HM* with a 90% probability. Pitch adjusting decision for $x_i^{nh}$ as follow:

$$x_i^{nh} \rightarrow \left\{ Yes\ if\ \ rn \leq PAR, \quad No\ if\ \ rn > PAR \right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.3)$$

A random number (*rn*) uniformly distributed over the interval [0,1] is generated for $x_1^{nh}$\`. If this random number is less than the *PAR*, $x_i^{nh}$ is replaced with its neighbour steel section in the section list. If this random number is not less than *PAR*, $x_i^{nh}$ remains the same. The selection of neighbour section is determined by neighbouring index. A *PAR* of 0.45 [13] indicates that the algorithm chooses a neighbour section with a 45%×HMCR probability. For example, if $x_i^{nh}$ is W14X68, neighbouring index is -2 or 2 and the section list is [W14X90, W14X82, W14X74, W14X68, W14X61, W14X53, W14X48], the algorithm will choose a neighbour one of the section (W14X82, W14X74 or W14X61, W14X53) with a 45%×*HMCR* probability, or remain the same section(W14X68) with a (100% - 45%)×*HMCR* probability. *HMCR* and *PAR* parameters are introduced to allow the solution to escape from local optima and to improve the global optimum prediction of the HS algorithm [20].

### 3.4.4 Update the harmony memory

If the new harmony $\left[x^{nh}\right]=\left\lfloor x_1^{nh}, x_2^{nh}, \ldots\ldots\ x_{ng}^{nh}\right\rfloor$ is better than the worst design in the *HM*, the new design is included in the *HM* and the existing worst harmony is excluded from the *HM*.

### 3.4.5 Termination criteria

Steps 3.4.3 and 3.4.3 are repeated until the termination criterion is satisfied. In this thesis, two termination criteria are used for HS. The first one stops the algorithm when a predetermined total number of searches (i.e. total number of iterations) are performed. The second criterion stops the process before reaching the maximum search number, if lighter frame is not found during a definite number of searches in HS. If one of these criteria is satisfied, the algorithm is terminated and the current optimum is defined as the final optimum design. Detailed flow charts for the HS with discrete design variables as shown in Figure 3.3.

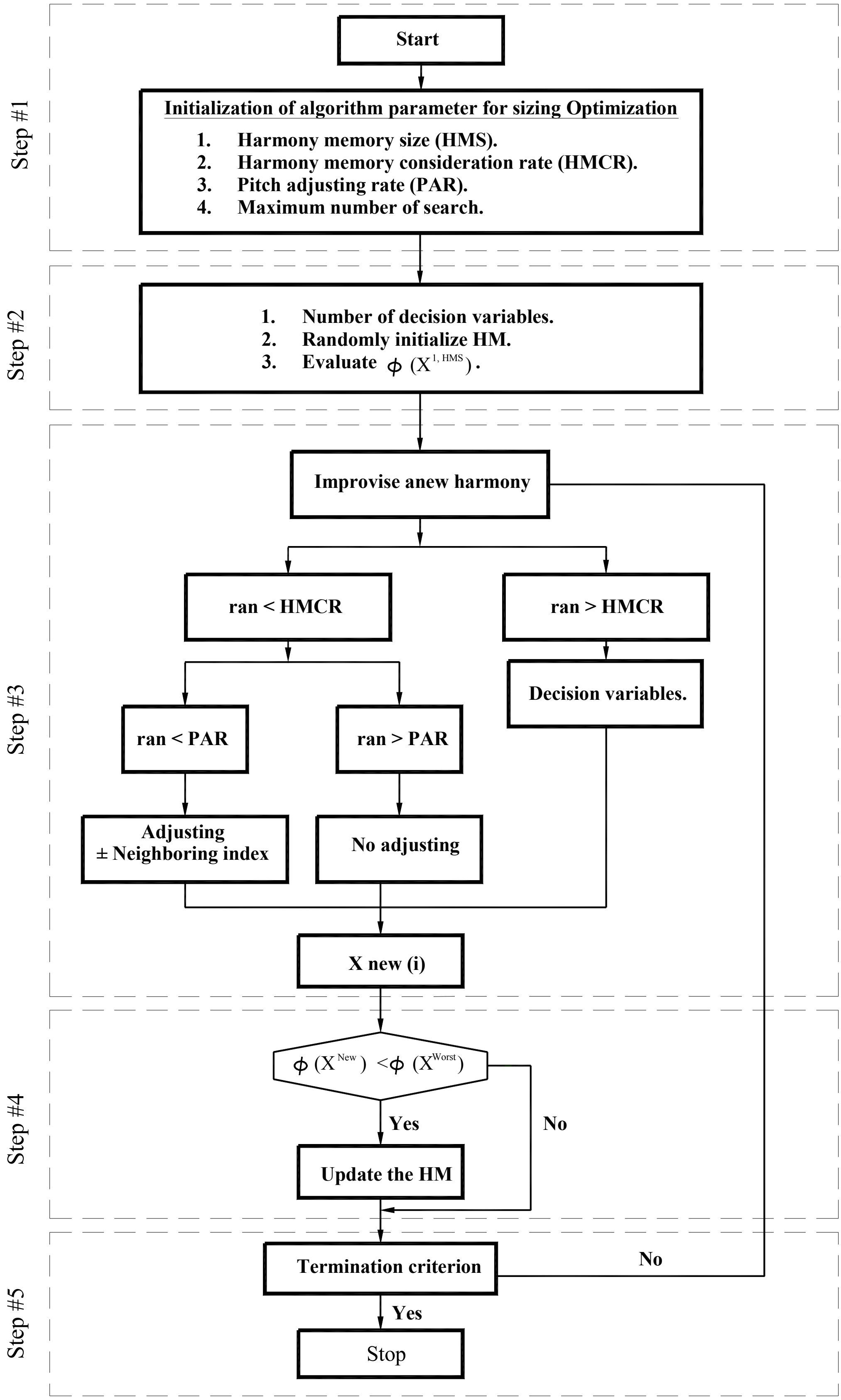


**Figure 3. 3: Flow-charts for the HS with discrete design variables.**

## 3.5 Comparison between the harmony search algorithm and other optimization techniques.

### 3.5.1 Harmony search (HS) and genetic algorithm (GA).

1. HS generates a new design considering all existing designs, while GA generates a new design from a couple of chosen parents by exchanging the artificial genes.
2. HS takes into account each design variable independently. On the other hand, GA considers design variables depending upon building block theory [47].
3. HS does not code the parameters, whereas GA codes the parameters. That is, HS uses real value scheme, while GA uses binary scheme (0 and 1).

### 3.5.2 Harmony search (HS) and simulating annealing (SA).

1. HS obtains a new design considering all existing designs as mentioned above, while SA generates a new design considering few neighbour designs of current design.
2. HS preserves better designs in its memory whereas SA does not have memory facility.

### 3.5.3 Harmony search (HS) and ant colony optimization (ACO).

1. ACO develops new designs considering the collective information obtained from the pheromone trails of ants, while HS develops the new designs considering the former designs stored in its memory, similar to ACO, but it also takes into account all the design variable databases with a predetermined probability. This facility provides a chance to improve the design by the values not stored in HS memory.
2. Local search process is applied to each other design with a predetermined probability in the HS, whereas ACO uses local search for only some elite designs.
3. HS updates its memory after each design is generated. On the other hand, ant colony is updated after as many designs as the numbers of ants in the colony are performed.

These differences provide a more effective and powerful approach for HS than GA, SA and ACO. For the HS superiority to be proven, two steel frames with rigid and semi-rigid connections are presented in this study. The two frames are also investigated by Kameshki and Saka (2003) using Genetic Algorithm. Moreover the effectiveness and robustness of harmony search algorithm, in comparison with genetic algorithm (GA) optimization were also studied.

# CHAPTER 4 : MODELLING OF STEEL FRAME STRUCTURS

## 4.1 Introduction.

In partially restrained frames, one of the most critical analysis steps is the modelling process. The modelling of any structure begins with an accurate representation of its members and components. The most difficult part of structural analysis is developing an accurate model that will correctly represent the structural system. In many cases, it is impossible to represent any building exactly with a model without making some general assumptions. For instance, structural materials are assumed to deform according to basic mechanics of materials. This assumption is reasonable for modelling purposes but in actuality may deviate due to weather conditions, construction and the actual consistency of the material. In developing a model, there are different levels of precision that can be achieved. This usually depends largely on the complexity of the structure, time allocated for design, cost of engineering and the uniqueness of the geometry or loads.

## 4.2 Modelling of steel frame structures with ANSYS.

Some of the basic frame analysis methods such as slope deflection, moment distribution, stiffness and flexibility methods can be modified to work with partially restrained connections but tend to be very tedious and complicated. Because most structural engineering use computers in the analysis of frames, there are several software packages designed to analyze structures such as SAP2000 and STAAD. The problem is that they cannot represent partially restrained connection behaviour with moment-rotational curve.

In this study, ANSYS software was used to model various elements and connection of steel structures. ANSYS is powerful in representing the partially restrained connections with a non-linear spring element. Also, ANSYS is used as its reason for second-order behaviour is evaluated accurately for partially restrained frames.

### 4.2.1 ANSYS package.

The ANSYS [48] program has a comprehensive graphical user interface (GUI) that gives users easy and interactive access to program functions, commands, documentation, and reference material. An intuitive menu system helps users navigate through the ANSYS program. Users can input data using a mouse, a keyboard, or a combination of both.

ANSYS finite element analysis software enables engineers to perform the following tasks:

- Build computer models or transfer CAD models of structures, products, components, or systems.
- Apply operating loads or other design performance conditions.
- Study physical responses, such as stress levels, temperature distributions, or electromagnetic fields.
- Optimize a design early in the development process to reduce production costs.

### 4.2.2 Elements library.

The element types are selected from the software based on the expected behaviour of members in frame.

#### 4.2.2.1 Beam-column element (BEAM3).

BEAM3 is a uniaxial element with tension, compression, and bending capabilities. The element has three degrees of freedom at each node: translations in the nodal x and y directions and rotation about the nodal z-axis. The characteristics of BEAM3 are as follows:

- The beam element must lie in an X-Y plane and must not have a zero length or area.
- The beam element can have any cross-sectional shape for which the moment of inertia can be computed. However, the stresses are determined as if the distance from the neutral axis to the extreme fiber is one-half of the height.
- The element height is used only in the bending calculations.
- The moment of inertia may be zero if large deflections are not used.

#### 4.2.2.2 Non-linear spring element (COMBIN39).

COMBIN39 is a unidirectional element with nonlinear generalized force-deflection capability that can be used in any analysis. The element has longitudinal or torsional capability in 1-D, 2-D, or 3-D applications. The longitudinal option is a uniaxial tension-compression element with up to three degrees of freedom at each node: translations in the nodal x, y, and z directions. No bending or torsion is considered. The torsional option is a purely rotational element with three degrees of freedom at each node: rotations about the nodal x, y, and z axes. No bending or axial loads are considered.

The element is defined by two (preferably coincident) node points and a generalized force-deflection curve. The points on this curve (D1, F1, etc.) represent force (or moment) versus relative translation (or rotation) for structural analyses.

The basic procedures for modelling steel frame structure with ANSYS are described in the following points and in Figure 4.1 flow chart.

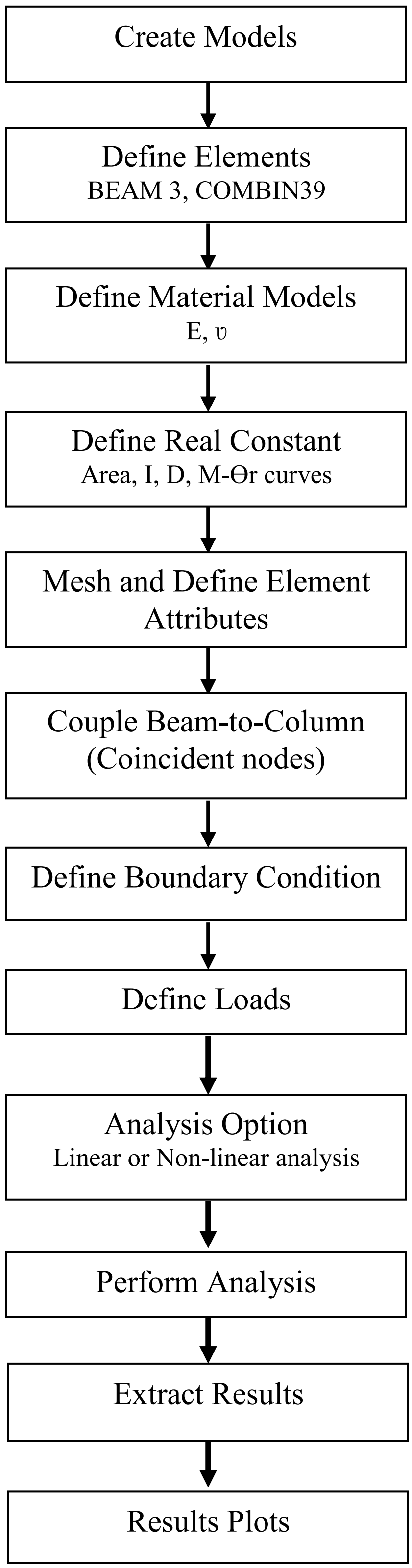


**Figure 4. 1: Flow chart for modelling steel frame structure with ANSYS.**

### 4.2.3 Modelling of steel connections.

The non-linear behaviour of the partially restrained connection, namely the moment rotation curve, is represented by the non-linear spring. By inputting the curve as data points to define the spring behaviour, the connection is represented. If the connection is very stiff the spring acts as a rigid joint while if the connection is very flexible the spring acts as a pin. The use of a spring in this situation allows for the representation of any connection more accurately then a typical pinned or rigid joint.

In the present study, the extend end plate connections without column stiffeners will be used and the semi-rigid connections are modeled with the Frye–Morris polynomial model [6, 7] as shown in equation (1).

$$\theta_r = c_1(KM)^1 + c_2(KM)^3 + c_3(KM)^5 \qquad \text{……...……………(1)}$$

Where $K$ is standardization constant depends upon connection type and geometry; $c_1$, $c_2$, $c_3$ are the curve fitting constants. The values of these constants are given in Table 4.1 [6, 7]. The values of the coefficients, such as the diameter of bolts, the gauge and the geometric dimensions used in the standardization constants are obtained by designing each connection in the frame during the optimum design cycles. Each design is carried out, with and without considering the geometric non-linearity according to the design problem.

**Table 4. 1: Curve fitting constants and standardization constant.**

| Connection types | Curve Fitting Constants Unit (in) | Standardization Constant Unit (in) |
|---|---|---|
| Extend end plate without column stiffeners | $C_1 = 1.83 \times 10^{-3}$<br>$C_2 = 1.04 \times 10^{-4}$<br>$C_3 = 6.38 \times 10^{-6}$ | $K = d_g^{-2.4}\, t_p^{-0.4}\, d_b^{-1.5}$ |

The non-linear analysis of steel frames takes into account both the geometrical non-linearity of beam-column members and non-linearity due to end connection flexibility of beam members. The columns of frames are generally continuous and do not have any internal flexible connections. However, the beams possess semi-rigid end connections, but have small axial forces with a geometric non-linearity of little importance. The geometry and size parameters of the extended end plate connections without column stiffeners as in Figure 4.2 [6, 7].

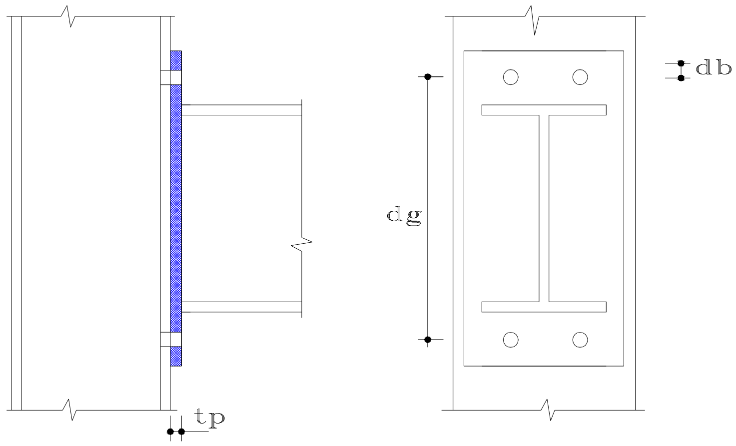


**Figure 4. 2: Extended end plate without column stiffeners.**

The semi-rigid connections used in the designs are same as Figure 4.2. Fixed and some oversized values for bolt size, gauge length and end plate thickness are selected during the design process so that they are safe and suitable for every design stage. The reason for employing such a fixed values for some of the connection size parameters is to shorten the computing time which is already very long due to design, Harmony search algorithm and non-linear analysis process.

On the other hand, the other connection size parameters such as beam height, the vertical distance between bolt groups (d, dg) are not fixed during the design process. They are calculated or selected depending on the standard steel section assigned to the beam throughout the design process. The connection size parameters which remain fixed during the optimum design process are given in Table 4.2 depending on the frame geometry.

**Table 4. 2: The fixed connection size parameter for all design models.**

| Model | Connection size parameters (in) | | |
|---|---|---|---|
| Ten-storey, one-bay | $t_p$ = 1 in | dg = d + 6 | $d_b$ = 1.125 in |
| Three-storey, two-bay | $t_p$ = 0.685 in | dg = d + 6 | $d_b$ = 1 in |

## 4.3 Geometric Nonlinearities

Structural analyses that include geometrical non-linearity’s are commonly termed second-order analyses. Geometrical non-linearity occur when members bend and the structure sway under loading. This additional displacement in the member causes second-order moments.

## 4.4 Material properties.

Material non-linearity, i.e. nonlinear stress-strain relationship, is a common cause of nonlinear structural behavior. Many factors can influence the material stress-strain properties, including load history (as in elastoplastic response), environmental conditions (such as temperature), and the amount of time that a load is applied (as in creep response). Stress-strain relationship can be classified as elastic, rigid plastic and elastic-plastic. For an elastic analysis, the stress-strain relationship is linear and the material never reaches its yield point. In rigid-plastic model, it is assumed that no deformation of the material takes place until the yield stress of the material has been reached. For elastic-plastic model, the material initially deforms elastically under increasing load and the stress-strain relationship is linear. The material becomes plastic when the yield stress of the material is reached. In this thesis used linear stress-strain relationship that agree with Kameshki and Saka (2003) [27].

## 4.5 Simulating of semi-rigid connection with SAP2000.

The problem is that they cannot represent partially restrained connection behaviour with moment-rotational curve, but we can simulate the connection using secant stiffness [49, 50] as shown in equation (2).

$$S = \frac{\Delta M}{\Delta \theta} \qquad \text{………..……………(2)}$$

**The following steps explain the procedures to obtain the secant stiffness value:**

1. Obtain a set of moment-rotation values using Frye-Morris polynomial model.
2. Draw moment-rotation relationship as shown in Figure 4.3.
3. Draw tangent # 1, at the ascending part of the curve.
4. Draw tangent # 2, at the peak of the curve.
5. Select the intersection point.
6. Draw down the rotation value and the moment value from the intersection point.
7. Finally, the new secant stiffness that describes the moment-rotation curve as mentioned in equation (2).

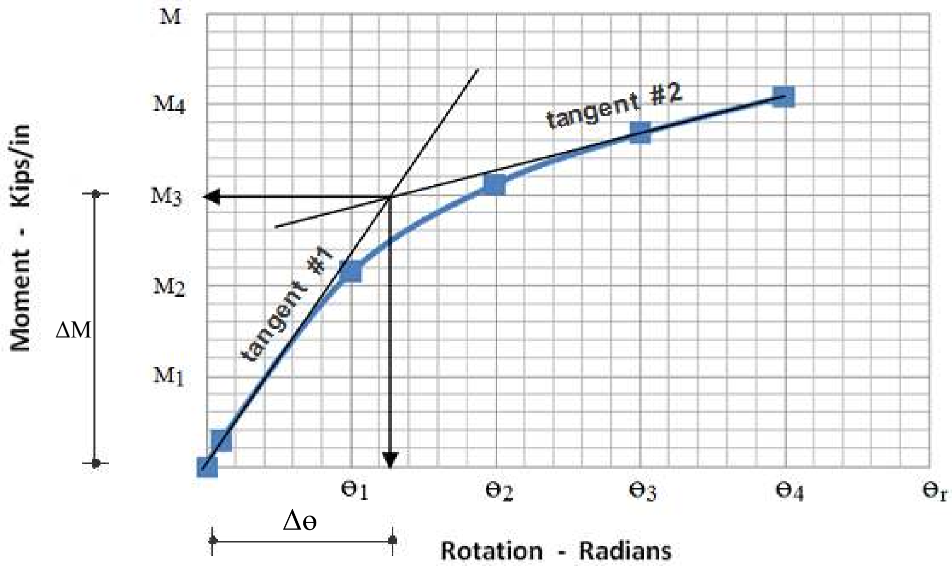


**Figure 4. 3: Moment – rotation curve.**

## 4.6 Comparison between ANSYS and SAP 2000 model.

In order to verify the results obtained by the ANSYS model, SAP software was used. The verification process was carried out using two models of a portal frame namely three-storey, two-bay frame and ten-storey, one-bay frame.

### 4.6.1 Semi-rigid steel frame of three-storey, two-bay model.

Three-storey, two-bay frame with semi-rigid connection loaded with uniformly distributed loads and horizontal loads as shown in Figure 4.4. Table 4.3 presents the section properties for the beam-column element are used. The elastic modulus, E, 30,000 ksi was assumed in the analysis and the Poison ratio, $\upsilon$, is 0.3.

**Table 4. 3: Section properties of three-storey, two-bay frame.**

| Type | Section | Area ( $in^2$ ) | Moment of Inertia, I ( $in^4$ ) | Depth, d ( in ) |
|---|---|---|---|---|
| Column group #1 | W12X35 | 10.3 | 285 | 12.5 |
| Column group #2 | W12X26 | 7.65 | 204 | 12.2 |
| Column group #3 | W8X24 | 7.08 | 82.7 | 7.93 |
| Column group #4 | W14X43 | 12.6 | 428 | 13.7 |
| Column group #5 | W12X30 | 8.79 | 238 | 12.3 |
| Column group #6 | W10X22 | 6.49 | 118 | 10.2 |
| Beam group #1 | W16X26 | 7.68 | 301 | 15.7 |

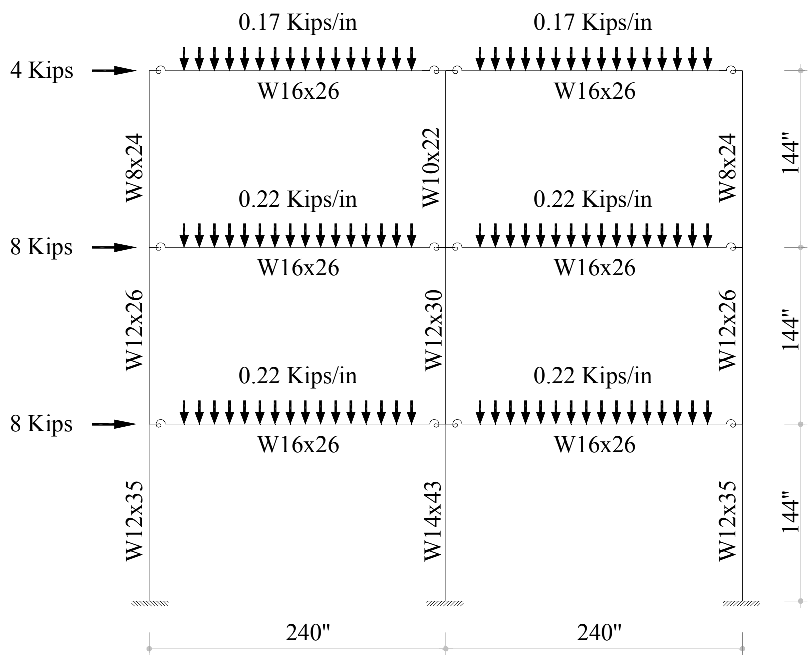


**Figure 4. 4: Three-storey, two-bay semi-rigid frame.**

To verify and validity of the model. The model was checked by another program such as SAP 2000 [51], with Non-linear analysis using the same geometry and loading.

The moment rotation curve was used as shown in Figure 4.5.

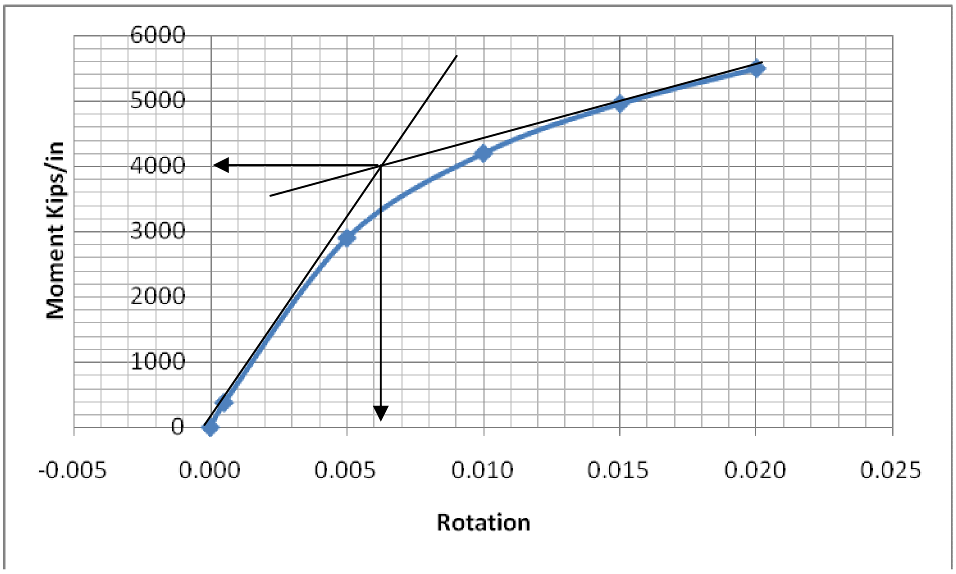


**Figure 4. 5: Moment-Rotation curve for beams.**

**Table 4. 4: The secant stiffness value for each beam-column connection.**

| Beam Group | Section | Secant Stiffness ( K.in/rad ) |
|---|---|---|
| Beam group #1 | W16X26 | $6.35x10^5$ |

The deformed shape and bending moment for Non-linear analysis, is shown in Figure 4.6, 4.7 respectively. A basic ANSYS input file as in Appendix-A.

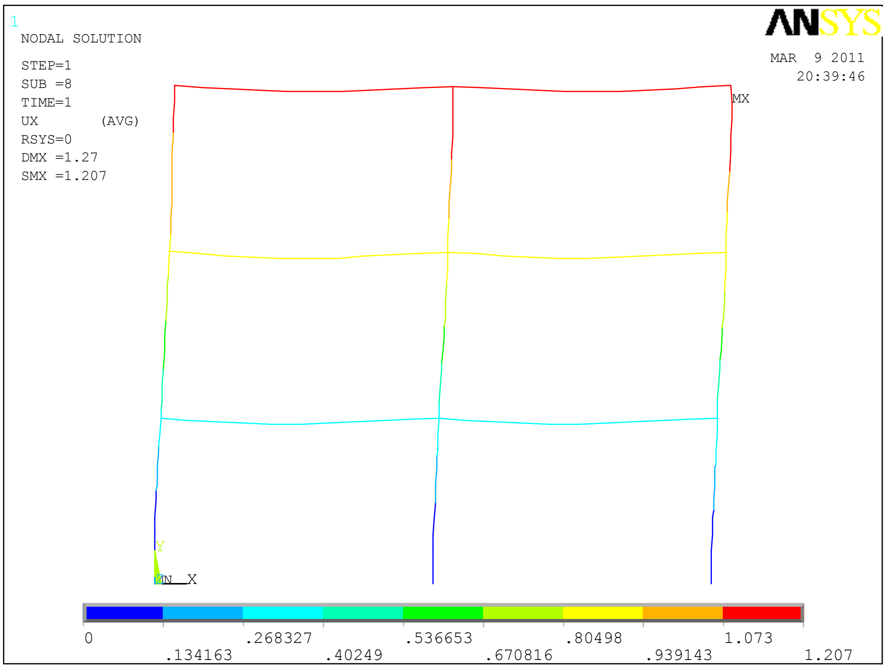


**Figure 4. 6: Three-storey, two-bay deformed shape.**

**Table 4. 5: Tabulates the comparison of horizontal displacements at the upper left corner and max bending moment at the column base with semi-rigid frame.**

| | Semi-Rigid Frame Connection | |
|---|---|---|
| | Non-Linear analysis | |
| | ANSYS 11 | SAP 2000, V14 |
| Upper left corner displacement (in) | 1.1929 | 1.21 |
| Max Base moment (K.in) | 912 | 905 |
| * 1 Kips = 4.45 KN & 1in = 25.4 mm | | |

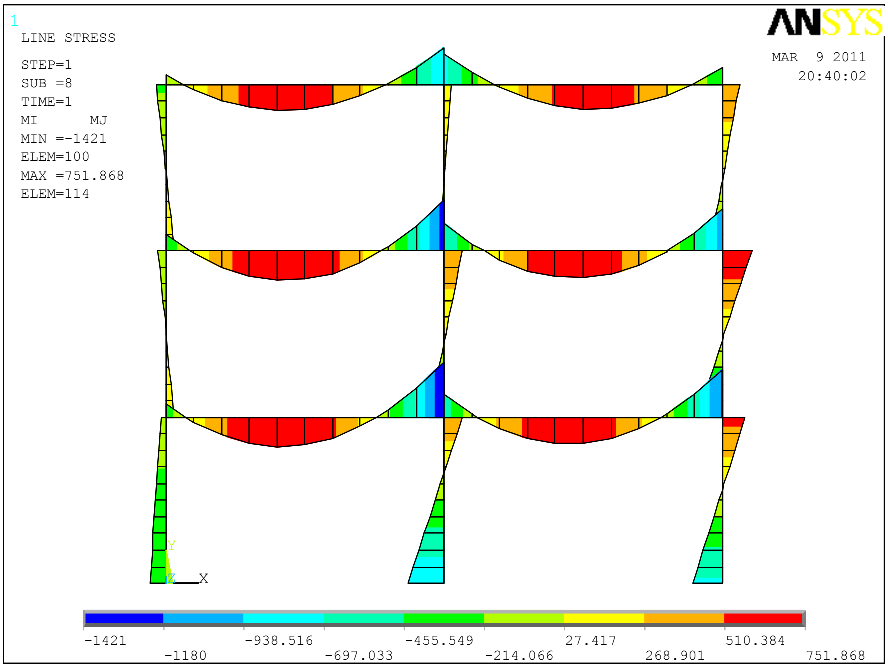


**Figure 4. 7: Three-storey, two-bay bending moment.**

### 4.6.2 Semi-rigid steel frame of ten-storey, one-bay model.

Ten-storey, one-bay frame with semi-rigid connection loaded with uniformly distributed loads and horizontal loads as shown in Figure 4.8. Table 4.6 presents the section properties for the beam-column element are used.

**Table 4. 6: Section properties of ten-storey, one-bay frame.**

| Type | Section | Area ( $in^2$ ) | Moment of Inertia, I ( $in^4$ ) | Depth, d ( in ) |
|---|---|---|---|---|
| Column group #1 | W27X146 | 43.1 | 5660 | 27.4 |
| Column group #2 | W21X122 | 35.9 | 2960 | 21.7 |
| Column group #3 | W21X101 | 29.8 | 2420 | 21.4 |
| Column group #4 | W18X76 | 22.3 | 1330 | 18.2 |
| Column group #5 | W14X82 | 24 | 881 | 14.3 |
| Beam group #1 | W24X68 | 20.1 | 1830 | 23.7 |
| Beam group #2 | W24X68 | 20.1 | 1830 | 23.7 |
| Beam group #3 | W27X84 | 24.8 | 2850 | 26.7 |
| Beam group #4 | W21X62 | 18.3 | 1330 | 21.0 |

The model was checked by SAP 2000 [51], with linear analysis using the same geometry and loading.

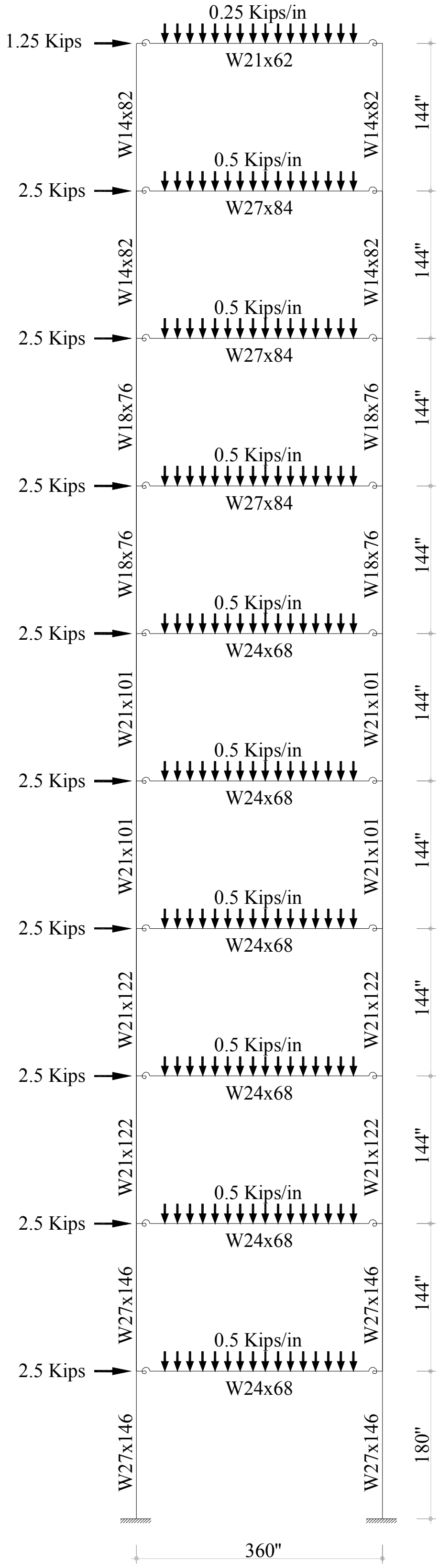


**Figure 4. 8: Ten-storey, one-bay semi-rigid frame.**

The moment rotation curve for each beam were used as shown in Figure 4.9.

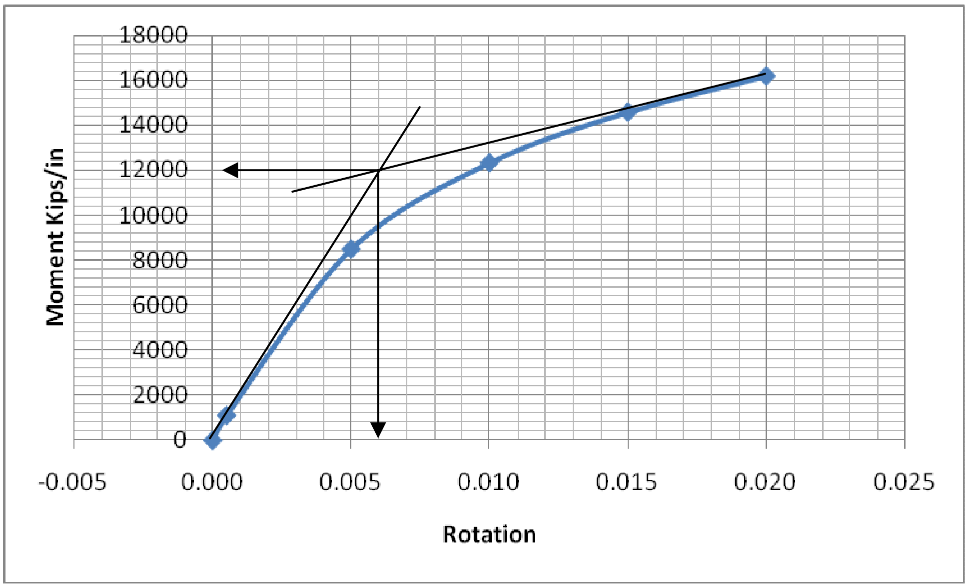


**Figure 4.9 1: Moment-Rotation Curve for beam #1, #2.**

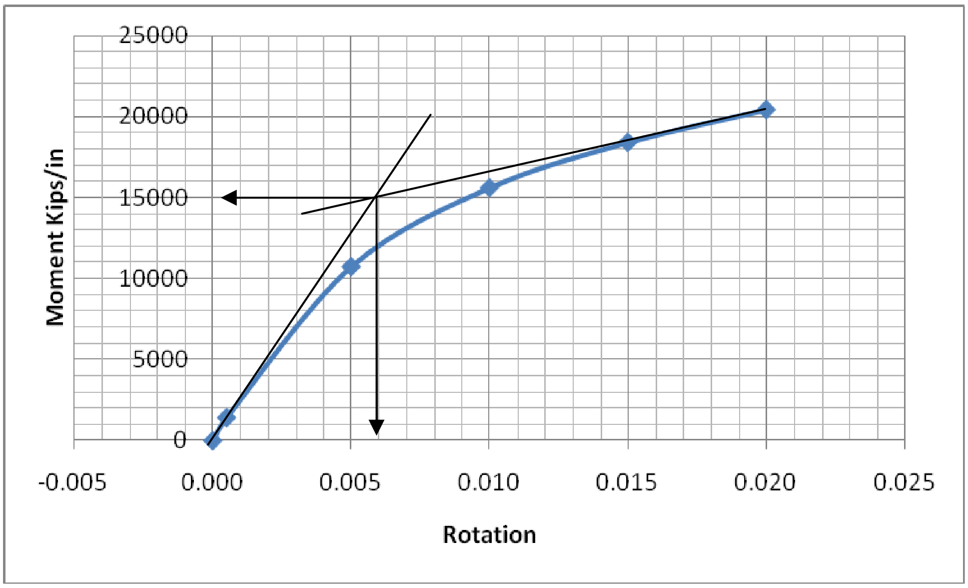


**Figure 4.9 2: Moment-Rotation Curve for beam #3.**

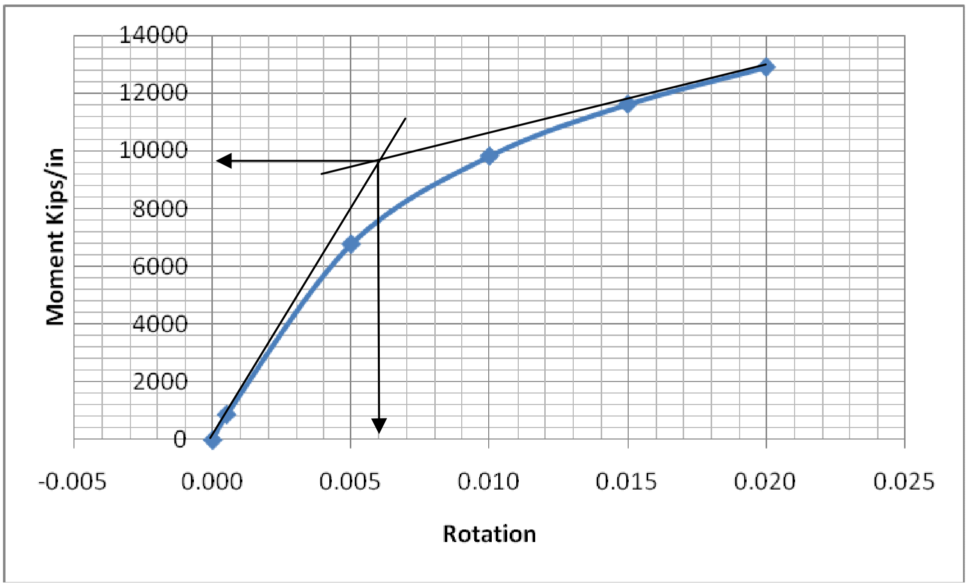


**Figure 4.9 3: Moment-Rotation Curve for beam #4.**

**Table 4. 7: The secant stiffness value for each beam-column connection.**

| Beam Group | Section | Secant Stiffness ( K.in/rad ) |
|---|---|---|
| Beam group #1 | W24X68 | $2x10^6$ |
| Beam group #2 | W24X68 | $2x10^6$ |
| Beam group #3 | W27X84 | $2.5x10^6$ |
| Beam group #4 | W21X62 | $1.5x10^6$ |

The deformed shape and bending moment for linear analysis, is shown in Figure 4.10, 4.11 respectively. A basic ANSYS input file as in Appendix-A.

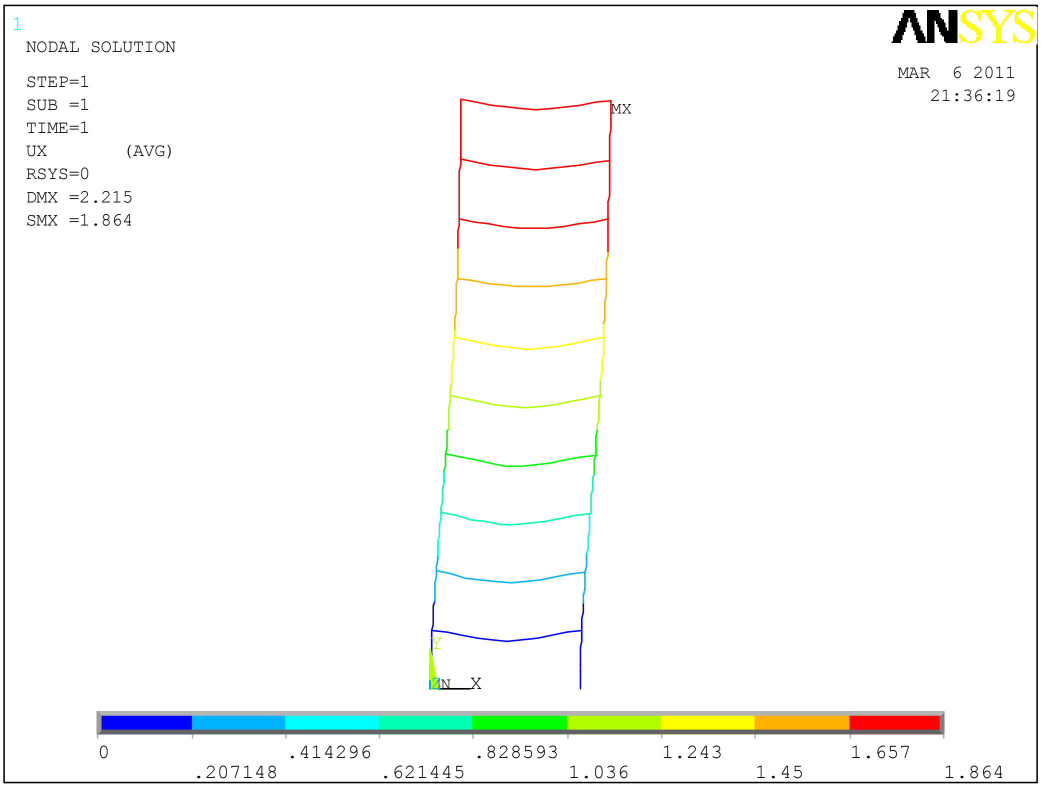


**Figure 4. 9: Ten-storey, one-bay deformed shape.**

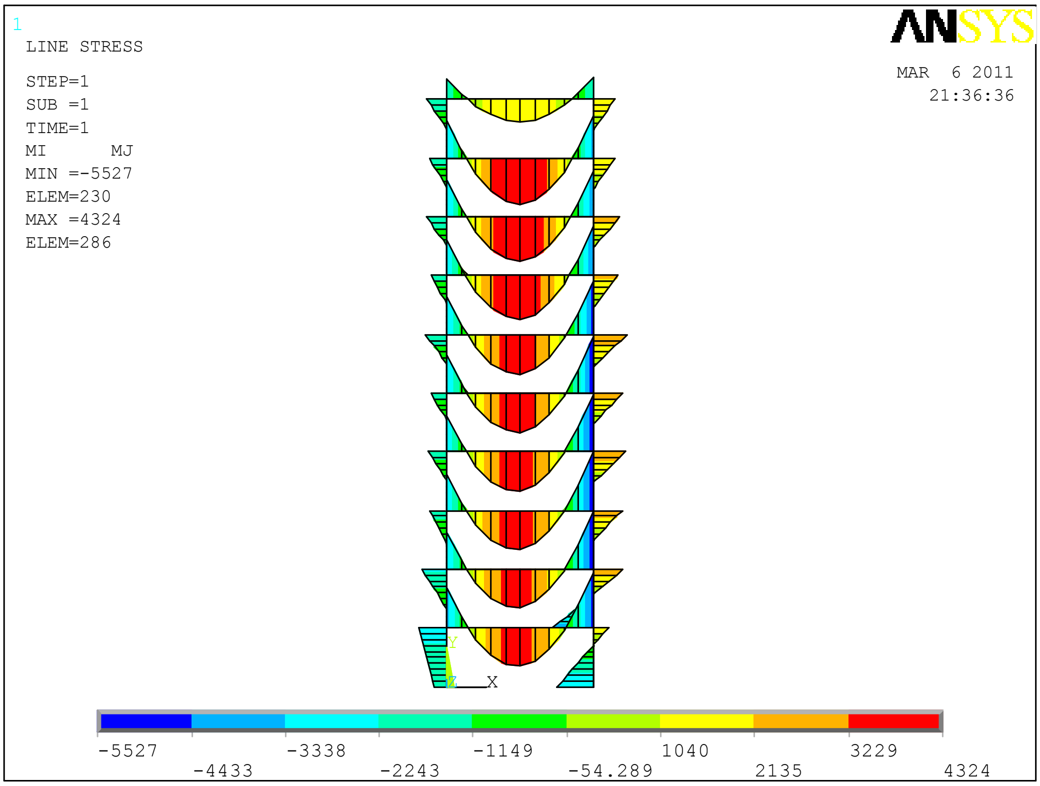


**Figure 4. 10: Ten-storey, one-bay bending moment.**

**Table 4. 8: Tabulates the comparison of horizontal displacement at the upper left corner and max bending moment at the column base with semi-rigid frame.**

| | Semi-Rigid Frame Connection | |
|---|---|---|
| | Linear analysis | |
| | ANSYS 11 | SAP 2000, V14 |
| Upper left corner displacement (in) | 1.8622 | 1.88 |
| Max Base moment (K.in) | 3170 | 3075 |
| * 1 Kips = 4.45 KN & 1in = 25.4 mm | | |

## 4.7 Concluding remarks.

It was an interesting result, which was obtained from SAP 2000 that compared with ANSYS model, that max sway at the upper left corner within 1.4%. The max bending moment at the column base vary from 1-3%.

The results indicate that the analysis and design of semi-rigid steel structure by sap 2000 became easy to use for the engineers.

# CHAPTER 5 : FORMULATION OF THE OPTIMIZATION PROBLEM

## 5.1 Introduction.

Formulation of an optimum design problem involves transcribing a verbal description of the problem into a well-defined mathematical statement. A set of variables to describe the design, called design variables, are given in the formulation. All designs have to satisfy a given set of constraints, which include limitations on material sizes, and response of the system. If a design satisfies all constraints, it is accepted as a feasible design. A criterion is needed to decide whether or not one design is better than another. This criterion is called the objective function. General flowchart diagram for optimum design could be sketched as shown in Figure 5.1 [52].

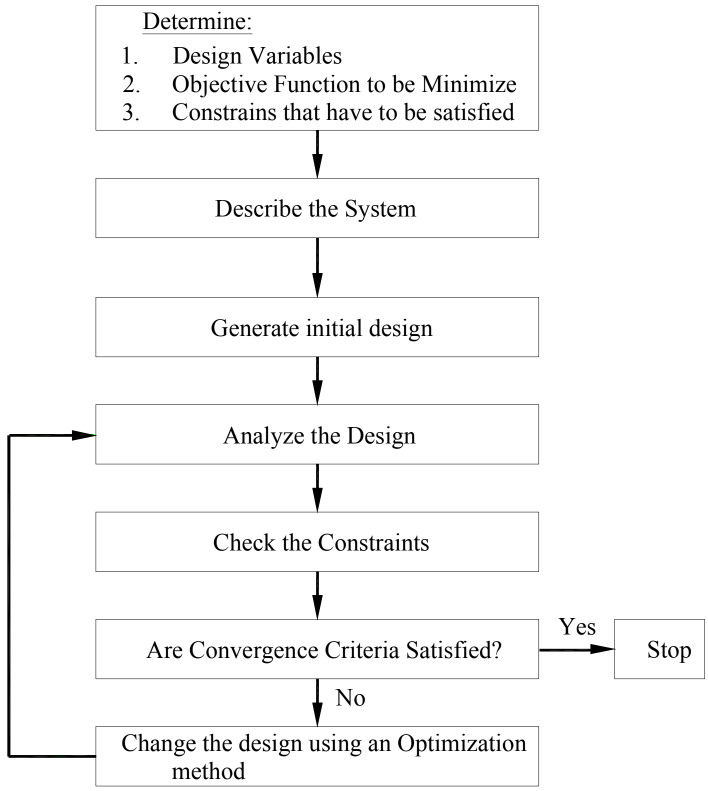


**Figure 5. 1: General flowchart diagrams for optimum design.**

## 5.2 Optimization problem and its formulation.

Design objectives that can be used to measure design quality include minimum weight, and maximum stiffness, as well as many others. Typically, the design is limited by constraints such as the choice of material, feasible strength, displacements, load cases, support conditions, and technical constraints (e.g., type and size of available catalog sections, etc.).

### 5.2.1 Objective function formula.

The minimum weight could be considered as the objective function, the standard steel sections are treated as design variables and the constraints are taken from the design codes. Therefore, the discrete optimum design problem of steel frames can be stated as follows.

$$Minimize \quad W(x) = \sum_{K=1}^{ng} A_k \sum_{i=1}^{mk} \rho_i L_i \qquad \ldots\ldots\ldots\ldots\ldots\ldots(5.1)$$

Subjected to the strength constraints of AISC-LRFD [1] and displacement constraints. In Eqn. (1), *mk* is the total numbers of members in group *k*, $\rho_i$ and $L_i$ are density and length of member *i*, $A_k$ is cross-sectional area of member group *k*, and *ng* is total numbers of groups in the frame.

### 5.2.2 Unconstrained objective function formula.

The unconstrained objective function $\varphi(x)$ is then written as follow.

$$\varphi(x) = W(x)[1 + KC]^{\varepsilon} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(5.2)$$

Where $C$= Constraint violation function, $K$ = Penalty constant, ε = Penalty function exponent. In this study K= 1.0, ε =2.0 [45].

### 5.2.3 Constraint violation function formula.

The constraint violation function is as follow.

$$C = \sum_{i=1}^{N_{jt}} C_i^t + \sum_{i=1}^{N_s} C_i^d + \sum_{i=1}^{N_{cl}} C_i^{sc} + \sum_{i=1}^{N_f} C_i^{sb} + \sum_{i=1}^{N_f} C_i^{db} + \sum_{i=1}^{N_c} C_i^I \qquad \ldots\ldots\ldots\ldots\ldots\ldots(5.3)$$

Where; $C_i^t$ is constraint violations for top-storey displacement, $C_i^d$ is constraint violations for interstorey displacement, $C_i^{sc}, C_i^{sb}$ is constraint violations for size constraints, $C_i^{db}$ is constraint violations for deflection and $C_i^I$ the interaction formulas of the LRFD specification; $N_{jt}$= number of joints in the top storey. $N_s$ and $N_c$= number of storey's except the top storey and number of beam columns, respectively. $N_{cl}$ = the total number of columns in the frame except the ones at the bottom floor. $N_f$ = number of storey. The penalty may be expressed as

$$C_i = \begin{cases} 0 & if \quad \lambda_i \le 0 \\ \lambda_i & if \quad \lambda_i > 0 \end{cases} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(5.4)$$

### 5.2.4 Displacement constraints.

The displacement constraints are

$$\lambda_i^t = \frac{|d_t|}{|d_t^u|} - 1.0 \leq 0 \quad i=1,\dots\dots\dots N_{jt} \qquad \dots\dots\dots\dots\dots\dots(5.5)$$

$$\lambda_i^d = \frac{|d_i|}{|d_t^u|} - 1.0 \leq 0 \quad i=1,\dots\dots\dots N_s \qquad \dots\dots\dots\dots\dots\dots(5.6)$$

Where $d_t$ : maximum displacement in the top storey, $d_t^u$ : allowable top storey displacement (Max height /300), $d_i$ : interstorey displacement in storey $i$, $di = (\sigma_n - \sigma_{n-1}$ / storey height), $d_i^u$ : allowable interstorey displacement (storey height /300).

### 5.2.5 Deflection constraints.

The deflection control for each beam is given as follows

$$\lambda_i^{db} = \frac{d_{db}}{d_{du}} - 1.0 \leq 0 \quad i=1,\dots\dots\dots N_f \qquad \dots\dots\dots\dots\dots\dots(5.7)$$

Where $d_{db}$ : maximum deflection for each beam.

- $d_{du}$ allowable floor girder deflection for service live load ≤ L/360.
- $d_{du}$ allowable floor girder deflection for service dead load and live load ≤ L/240.

### 5.2.6 Size constraints.

The size constraint employed for constructional reasons is given as follows

$$\lambda_i^{sc} = \frac{d_{un}}{d_{bn}} - 1.0 \leq 0 \quad i=1,\dots\dots\dots N_{cl} \qquad \dots\dots\dots\dots\dots\dots\dots(5.8)$$

$$\lambda_i^{sb} = \frac{d_{bf}}{d_{bc}} - 1.0 \leq 0 \quad i=1,\dots\dots\dots N_f \qquad \dots\dots\dots\dots\dots\dots\dots(5.9)$$

Where $d_{un}$ and $d_{bn}$ are depths of steel sections selected for upper and lower floor columns.

### 5.2.7 Strength constraints.

The strength constraints taken from AISC-LRFD [1] are expressed in the following equations. For members subject to bending moment and axial force.

$$for \quad \frac{P_u}{\phi P_n} \geq 0.20$$

$$\lambda_i^I = \left(\frac{P_u}{\phi P_n}\right) + \frac{8}{9}\left(\frac{M_{UX}}{\varphi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) - 1.0 \leq 0 \quad i=1,\dots\dots\dots N_c \qquad \dots\dots\dots\dots\dots\dots(5.10)$$

$$for\ \frac{P_u}{\phi P_n} < 0.20$$

$$\lambda_i^I = \left(\frac{P_u}{2\phi P_n}\right) + \left(\frac{M_{UX}}{\varphi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) - 1.0 \quad \leq \quad 0 \qquad i=1,\ldots\ldots\ldots N_c \qquad \ldots\ldots\ldots\ldots\ldots\ldots(5.11)$$

Where $P_u$ = requires axial strength (compression or tension), $P_n$ = nominal axial strength (compression or tension), $M_{ux}$ = requires flexural strengths about the major axis, $M_{uy}$ = requires flexural strengths about the minor axis, $M_{nx}$ = nominal flexural strength about the major axis, $M_{ny}$ = nominal flexural strength about the minor axis (for two-dimensional frames, $M_{uy}$ = 0), $\varphi = \varphi_c$ = resistance factor for compression (equal 0.85), $\varphi = \varphi_t$ = resistance factor for tension (equal 0.90), $\varphi b$ =flexural resistance factor (equal 0.90).

If the shape is compact, check for lateral- torisional buckling (LTB) as follows

1. $L_b \leq L_p$ , there is no LTB, and

$$M_{nx} = M_p = F_y Z_x \qquad \ldots\ldots\ldots\ldots\ldots\ldots(5.12)$$

2. $L_p < L_b \leq L_r$ ,there is inelastic LTB, and

$$M_{nx} = C_b \left[ M_p - (M_p - M_r)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots(5.13)$$

$$M_r = (F_y - F_r) S_x \qquad \ldots\ldots\ldots\ldots\ldots\ldots(5.14)$$

$$L_p = 1.76\, r_y \sqrt{\frac{E}{F_y}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(5.15)$$

$$L_r = \frac{r_y X_1}{(F_y - F_r)} \sqrt{1 + \sqrt{1 + X_2 (Fy - Fr)^2}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(5.16)$$

$$X_1 = \frac{\pi}{Sx}\sqrt{\frac{EGJA}{2}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(5.17)$$

$$X_2 = 4\frac{Cw}{Iy}\left(\frac{Sx}{GJ}\right)^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots(5.18)$$

Where $L_b$ = unbraced length, $L_p$ = unbraced length at the plastic moment, $L_r$ = unbraced length at the buckling moment, $M_p$= plastic moment, $F_y$= yield stress of steel, $Z_x$ = plastic section modulus, $C_b$ = moment coefficient, $M_r$ = buckling moment at $L_r$, $F_r$= compressive residual stress in flange: 10 ksi, $S_x$ = elastic section modulus about major axis, $r_y$ = : governing radius of gyration about minor axis, $E$ = modulus of elasticity of steel, $G$ = shear modulus of elasticity of steel, $A$= cross sectional area, $Cw$ = warping constant, $Iy$ = moment of inertia about Y- axis.

#### 5.2.7.1 Design strength of columns.

The AISC-LRFD [1] design strength of columns is computed as

$$P_n = A_g F_{cr} \qquad \text{.................(5.19)}$$

$$F_{cr} = 0.658^{\lambda_c^2} F_y \qquad 0 \le \lambda_c \le 1.5 \qquad \text{.................(5.20)}$$

$$F_{cr} = \frac{0.877}{\lambda_c^2} F_y \qquad \lambda_c > 1.5 \qquad \text{.................(5.21)}$$

$$\lambda_c = \frac{KL}{r\pi}\sqrt{\frac{F_y}{E}} \qquad \text{.................(5.22)}$$

Where $Ag$ = cross-sectional area of member, $Fcr$= critical compressive stress, $\lambda c$= column slenderness parameter, $Fy$= yield stress of steel, $K$=effective-length factor, $L$= member length, $r$ = governing radius of gyration, $E$= modulus of elasticity. The effective length factor $K$, for an unbraced frame is calculated from the following approximate equation taken from Dumonteil [53]. The out-of-plane effective length factor for each column member is specified to be $Ky$ = 1.0, while that for each beam member is specified to be $Ky$ = L/6 (i.e., floor stringers at L/6 points of the span). The length of the unbraced compression flange for each column member is calculated during the design process, while that for each beam member is specified to be L/6 of the span length.

$$K = \sqrt{\frac{1.6G_A G_B + 4.0(G_A + G_B) + 7.50}{G_A + G_B + 7.50}} \qquad \text{.................(5.23)}$$

Where subscripts $A$ and $B$ denote the two ends of the column under consideration. The restraint factor $G$ is stated as

$$G = \frac{\sum (Ic/Lc)}{\sum (I_B/L_B)} \qquad \text{.................(5.24)}$$

Where $I_c$ is the moment of inertia and $L_c$ is the unsupported length of a column section; $I_B$ is the moment of inertia and $L_B$ is unsupported length of a beam. Σ indicates a summation for all members connected to that joint ($A$ or $B$) and lying in the plane of buckling of the column under consideration.

Therefore, the beam stiffness $I_b/L_b$ given in (5.24) is multiplied by the factor of $1 / (1 + 6E\, I_b/L_b k)$ to consider semi-rigid end connections, where k is rotational spring stiffness of corresponding end [54].

#### 5.2.7.2 Design strength of beams.

Design strength of beams is $\varphi_b M_n$ . As long as $\lambda \leq \lambda_p$, the $M_n$ is equal to $M_p$ and the shape is compact. The plastic moment $M_p$ is calculated from the equation

$$M_p = F_y \, z_x \qquad \text{..................(5.25)}$$

Where $Z$= the plastic section modulus, $\lambda_p$= slenderness parameter to attain $M_p$. Details of the formulations are given in the AISC-LRFD [1]. Gaylord et al. [55] and Galambos et al. [56] can also find broad information in the books.

## 5.3 Development of harmony search optimization algorithm.

### 5.3.1 Harmony memory size.

The harmony memory size (*HMS*) was selected depending on the geometric of the structure. *HMS* is also sensitive to the number of design variables. When the number of design variables is increased, the search space enlarges.

### 5.3.2 Harmony memory consideration rate.

The harmony memory consideration rate (*HMCR*) is also sensitive. A value of 1.0 for *HMCR* is not appropriate because of 0% possibility that the new design may be improved by values not stored in the *HM*.

### 5.3.3 Pitch adjusting rate.

HS is also influenced by the value of pitch adjusting rate (*PAR*) which was taken as 0.45. Using higher values for *PAR* caused non-optimal designs, while lower values for it resulted in local optima. The neighboring index used in the pitch adjustment selected as ±2 depends on the geometry of the structure.

### 5.3.4 Maximum number of searches.

The maximum number of searches is another important parameter in the HS algorithm.

### 5.3.5 Random number.

A random number (*rn*) uniformly distributed over the interval [0,1] is generated.

### 5.3.6 Generation of harmony.

The *HM* matrix is filled with randomly generated designs as the *HMS*. Each row of harmony memory matrix contains the values of design variables (w-section) which are randomly selected feasible solutions from the design pool. Hence, this matrix has n columns where n is the total number of design variables and *HMS* rows which is selected in the first step. *HMS* is similar to the total number of individuals in the population matrix of the genetic algorithm.

### 5.3.7 Finite element analysis.

ANSYS software used to analyze the structure with linear or non-linear analysis according to the optimization problem.

### 5.3.8 Unconstrained objective function.

The unconstrained objective function calculates the weight of new design that including penalty if any constraint not satisfy.

### 5.3.9 Generation of a new harmony.

If the new harmony is better than existing worst harmony in the *HM*, new harmony is included in the *HM* and the existing worst harmony is excluded from the *HM*. The process is repeated until the best harmony is obtained. Detailed flow charts for the optimum design algorithm using HS as shown in Figure 5.2.

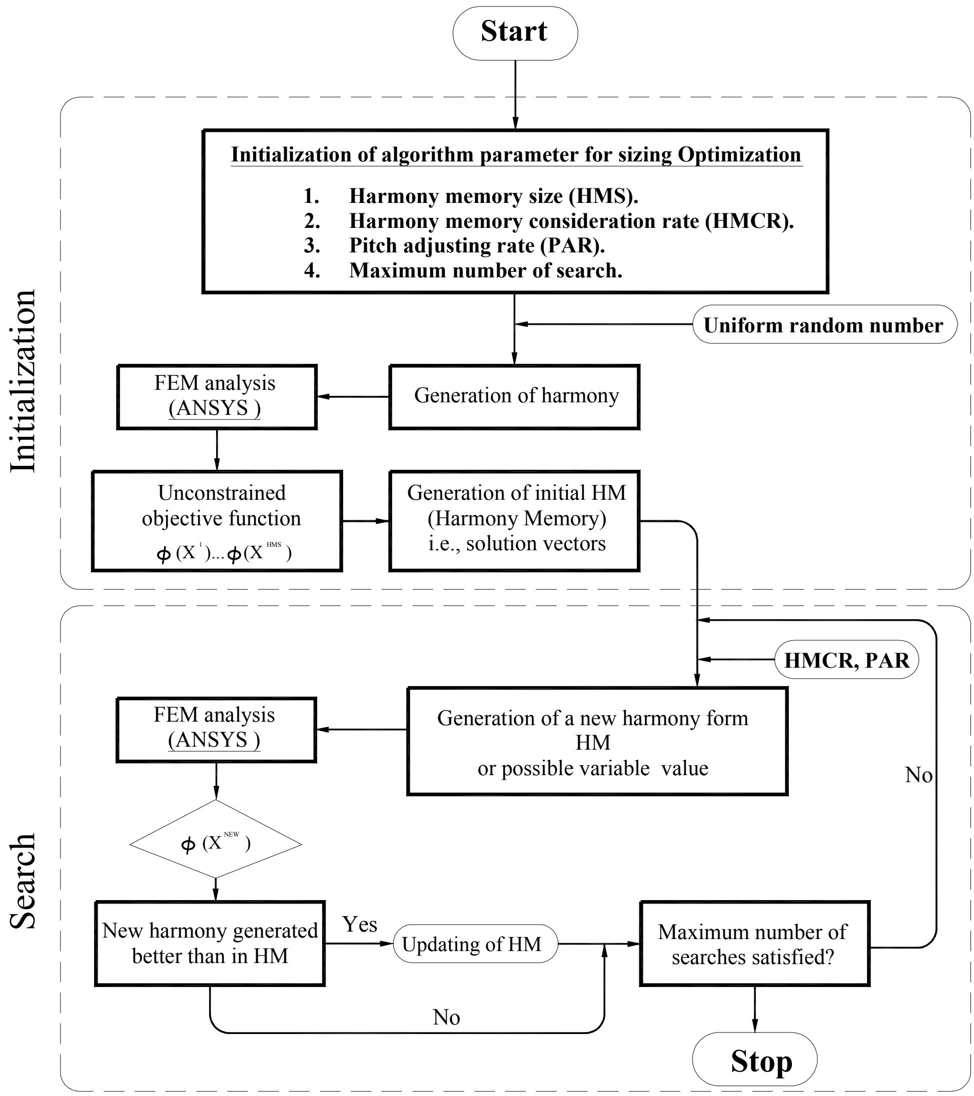


**Figure 5. 2: Harmony search algorithm optimization procedure.**

Detailed flow charts for the initialization process using ANSYS-MATLAB as shown in Figure 5.3.

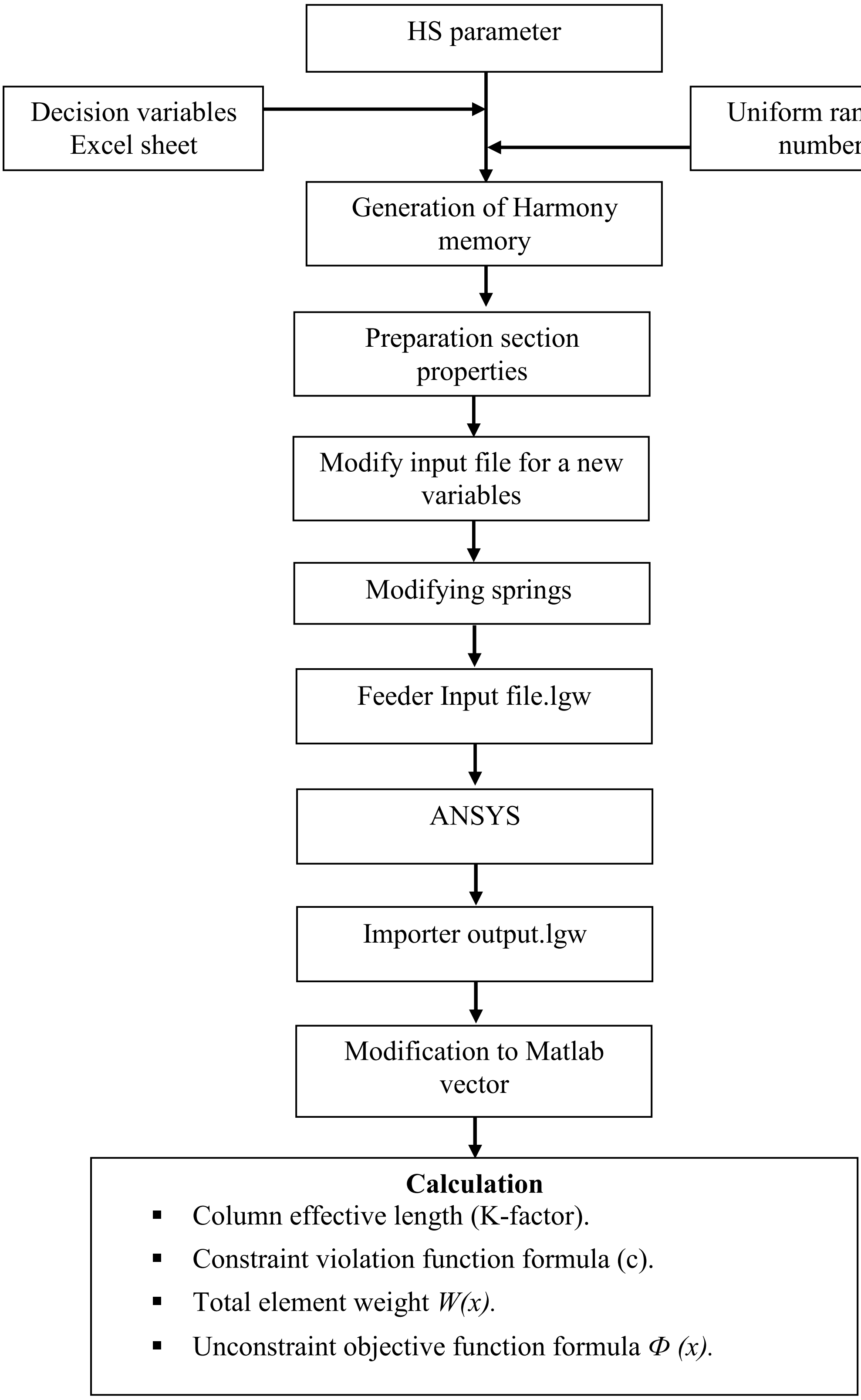


**Figure 5. 3: Detailed flow charts for the initialization process.**

## 5.4 ANSYS – MATLAB batching file.

- - b : Open ANSYS software.
- - i : Input data to ANSYS and solve the model (Input.lgw).
- - o : Output data (Output.lgw).

## 5.5 Steel section catalog used in this study.

Two catalogs are used in this study as the following:

### 5.5.1 Full Catalog Section (FCS).

This catalog contain all beam-column members with 168 W sections (W40 to W8) with weight less than 200 Ib/ft. as shown in Appendix-B.

### 5.5.2 Selected Catalog Section (SCS).

This catalog contains two section lists comprised 168 W sections each are used in the design.

- The first one is column catalog with the height/width ratio less than 2 (number of column equal 93 w section) with weight less than 200 Ib/ft. as shown in Appendix-B.
- The second one is beam section list with the height/width ratio greater than 2 (number of beam equal 75 w section) with weight less than 200 Ib/ft. as shown in Appendix-B.

# CHAPTER 6 : ANALYSIS RESULTS AND DISCUSSION

## 6.1 Introduction.

The harmony search optimization algorithm adopted in this thesis is used to obtain a steel frame with minimum weight by selecting a set of standard steel sections which are light but yet strong enough to carry the imposed loads. The Constraints taking into account while developing the main optimization criteria are: Strength constraints of AISC-LRFD specification, displacement constraints and size constraints for beam-columns elements. [1].

For the HS superiority to be proven, two steel frames with rigid and semi-rigid connections are presented in this study. The two frames are also investigated by Kameshki and Saka (2003) using Genetic Algorithm [27]. Moreover the effectiveness and robustness of harmony search algorithm, in comparison with genetic algorithm (GA) optimization were also studied.

## 6.2 Optimization of three-storey, two-bay steel framed structure.

A three-storey, two-bay steel frame structure optimization using HS search algorithm is presented in this chapter using various assumptions, in order to compare the results of the HS algorithm with results of an identical structure being optimized using the Genetic Algorithm Optimization Technique (GA). The structure has been analyzed assuming rigid beam-to-column connections and then another analysis has been carried out assuming semi-rigid connections using the Full Catalog Section (FCS) and Selected Catalog Section (SCS). The analysis has been run twice for each of the previously mentioned assumptions, once considering a linear behavior and then assuming a non-linear behavior. Finally, the results of all analysis have been compared to those in its GA counterpart. The structure being optimized is shown in Figure 6.1.

**The design constant parameters which used are listed:**

- Young's modulus of the steel $E$ =30,000 ksi.
- Yield stress of $Fy$ =36 ksi.
- Allowable top storey sway (H/300) = 1.44 in.
- Allowable interstorey sway (h/300) = 0.48 in.
- Allowable deflection for service dead and live load (L/240) = 1 in.
- The member effective length factors $Kx$ is calculated from the approximate equation proposed by Dumonteil [53] as in equation (6.1).

$$Kx = \sqrt{\frac{1.6G_A G_B + 4.0(G_A + G_B) + 7.50}{G_A + G_B + 7.50}} \quad \text{..................(6.1)}$$

- The out-of-plane effective length factor for each column ($Ky$) = 1.0.
- The out of plane unbraced length for each beam member was specified to be L/6 = 40 in.

**The optimization constant which used are listed:**

- Penalty constant of $K$ = 1.0 [45].
- Penalty function exponent of $\varepsilon$ = 2 [45].

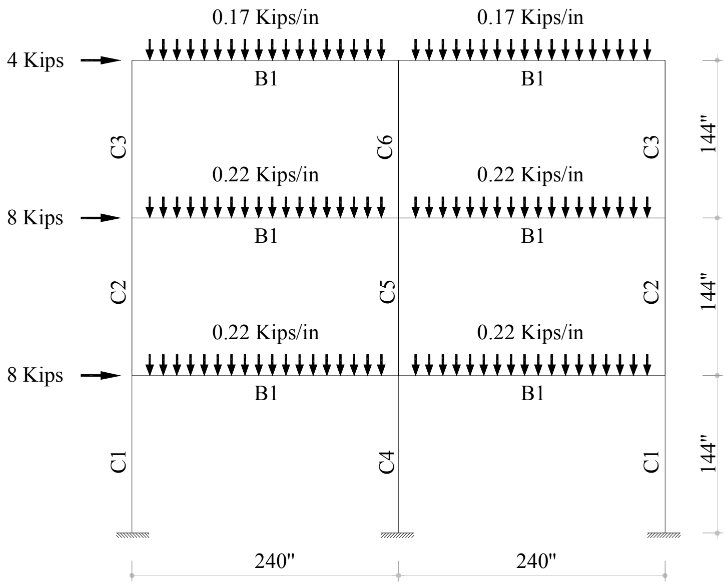


**Figure 6. 1: Three-storey, two-bay frame.**

For the HS algorithm not to be stuck in local optimum solutions, and to encourage the HS algorithm to search the global spectrum of solutions, a set of very-carefully-selected parameters have been established by various trials, and recommendations found in the literature review.

The obtained values of the HS tuning parameter are as follows:

1. **Harmony memory size (*HMS*).**

   The harmony memory size *(HMS)* was selected depending on the geometric of the structure. *HMS* is also sensitive to the number of design variables. After several computational trials, Harmony memory size *(HMS)* found to be equal 15.

2. **Harmony memory consideration rate (*HMCR*).**

   The chosen value of HMCR as 0.9 reflects the confidence of the author, because the design variables (steel sections) are less than 200 Ib weight as mentioned in Appendix–B (Full Catalog Section FCS and Selected Catalog Section SCS).

3. **Pitch adjusting rate (*PAR*).**

As *PAR* is very sensitive parameter as it takes the optimization problem from local to global. Therefore, the *PAR* is taken 0.45 which agrees with [13].

4. **Neighbouring index used in the pitch-adjustment.**

By determining *HMCR* and *PAR* values and after many trials from ± 1 to ±3, the solution is completely improved with 500 iterations. This needs lots of iteration that may reach 5000 iterations. However, the solution is obviously improved by ±2 value. Moreover, the number of iterations decreased steadily and this needs less time to find the optimal solution.

5. **Termination criterions.**

After completely the first termination of 1000-th iterations, if the optimal solution doesn't reach the ultimate path, the solution itself can be stopped automatically because it time wasting and the main reason is the random selection of the design variables (steel sections). On the one hand, the maximum number of iterations is selected to be 2500 because the solution after 1000-th iterations converges slightly to the optimum solution.

6. **Numbers of independent runs.**

Ten independent runs are made to minimize the weight of the steel frames with rigid and semi-rigid connections.

### 6.2.1 Optimization by analyzing the connection as rigid frame.

Ten independent optimum frame designs are achieved using Full Catalog Section (FCS) selection as in Appendix-B; analyzed using geometric non-linearity. The result is presented in Table 6.1.

Depending on the results of the ten independent runs are presented in Table 6.1. It is observed that HS converged to the optimum designs between 1155-th and 2235-th iteration. HS develops the optimum design at 2235-th iterations and remains unchanged until the maximum number of iterations is obtained 2500-th. The average weight of ten different designs is 6820 lb, with a standard deviation of 203 lb. The maximum sway corresponding to the optimum design is 0.64 in which smaller than the allowable limit by AISC-LRFD-1.44 in.

**Table 6. 1: Optimum design results in the three-storey, two-bay frame with rigid connections (FCS).**

| Three-Storey, two-bay frame | | |
|---|---|---|
| | **Rigid Connection (FCS)** | |
| **Frame Analysis** | **Optimum Weight** | **Max Improvisation** |
| **1** | 6576 | 1392 |
| **2** | 6864 | 2130 |
| **3** | 6792 | 2235 |
| **4** | 6888 | 2133 |
| **5** | 6792 | 1586 |
| **6** | **6528** | **2235** |
| **7** | 6792 | 1803 |
| **8** | 6768 | 1621 |
| **9** | 6924 | 1600 |
| **10** | 7272 | 1155 |
| **Min ( Ib )** | **6528** | |
| **Average ( Ib )** | **6820** | |
| **Standard Deviation ( Ib )** | **203** | |

Figure 6.2 present the optimum design history (weight verses number of iterations). It proves that after 1000-th iterations the minimum weight slows down and become unchanged.

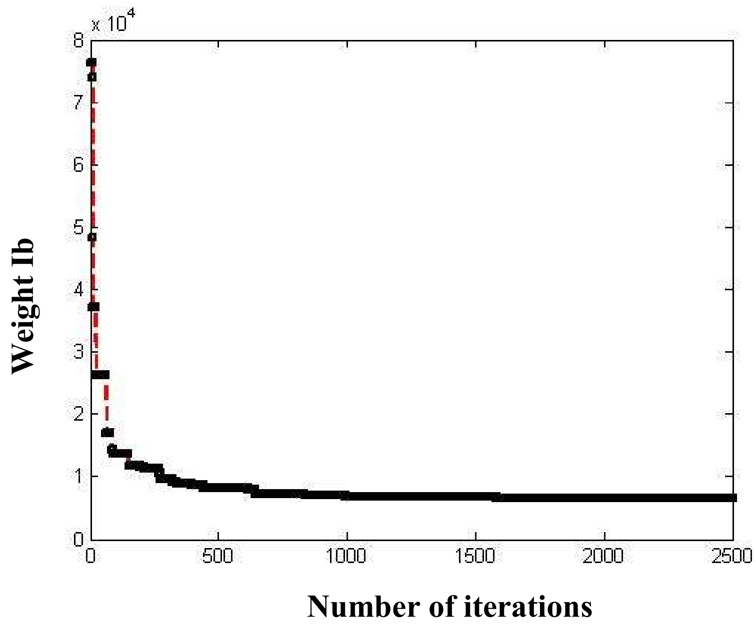


**Figure 6. 2: Optimum design history of three-storey, two-bay rigid frame (FCS).**

### 6.2.2 Optimization by analyzing the connection as semi-rigid frame.

The previous problem is also solved using semi-rigid connections. After studying all types of connections extensively, the choice has been made on the extended end plate without column stiffeners to be used as a connection since it's the most common connection, and has a lower cost, and requires no extraordinary skills in its application. To model such a connection, several mathematical models were studied. Frye–Morris polynomial model is used because it gives a powerful result that represents the moment–rotation curve as clarified in equation (6.2). The fixed value and geometric parameter was discussed chapter 4 section 4.2.3.

$$\theta_r = c_1(KM)^1 + c_2(KM)^3 + c_3(KM)^5 \qquad \text{……...…………(6.2)}$$

In this study, the structure is analyzed using semi rigid connection with FCS and SCS. The results of both analysis (FCS & SCS) are compared in the following sections.

#### 6.2.2.1 Optimization results using FCS.

A three-storey, two-bay steel frame with the same geometric and loading is shown in Figure 6.1. The results of ten independent runs are obtained as in Table 6.2.

**Table 6. 2: Optimum design results of the three-storey, two-bay frame with semi-rigid connection (FCS).**

| Three-Storey, two-bay frame | | |
|---|---|---|
| | Semi-rigid Connection (FCS) | |
| **Frame Analysis** | **Optimum Weight** | **Max Improvisation** |
| **1** | 6516 | 1622 |
| **2** | 6648 | 1204 |
| **3** | 6504 | 1751 |
| **4** | 7128 | 1622 |
| **5** | 6372 | 1135 |
| **6** | 6396 | 1490 |
| **7** | 6396 | 1135 |
| **8** | 6696 | 2091 |
| **9** | 6348 | 1255 |
| **10** | **6300** | **2366** |
| **Min ( Ib )** | **6300** | |
| **Average ( Ib )** | **6530** | |
| **Standard Deviation ( Ib )** | **246** | |

The minimum weight using semi-rigid connections 6300 Ib is less than the minimum weight using the rigid one 6528 Ib, even though both of them has the same catalog (FCS), moreover the rigid connection is more expensive than the semi-rigid one. In addition, the results showed that the maximum sway is 0.63 inch in case of rigid frame which is less than 0.93 inch in case of semi-rigid frame due to reduction of stiffness.

The optimum design history is shown in Figure 6.3. It is observed from the Figure; the frame weight starts to decline in the first 1400-th iterations, but the weight become flat after 1400-th iterations.

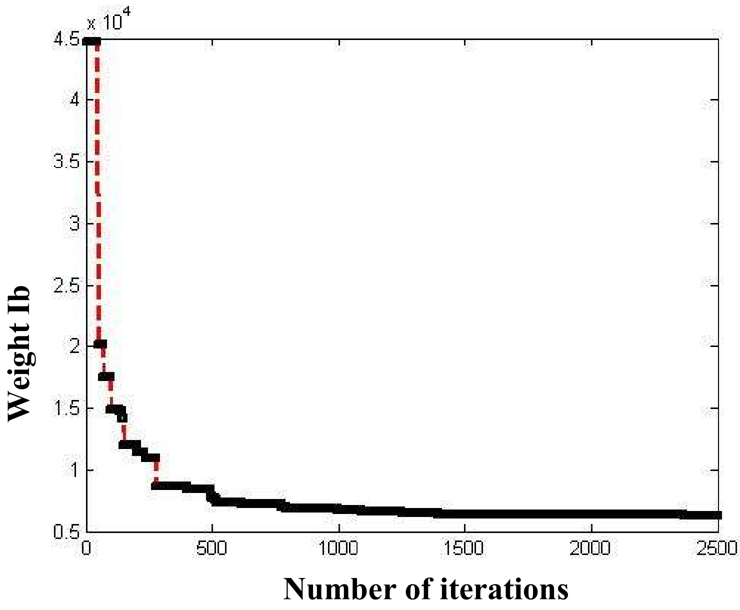


**Figure 6. 3: Optimum design history of three-storey, two-bay semi-rigid frame (FCS).**

### 6.2.2.2 Optimization results using SCS.

Selected Catalog Section (SCS) is going to be used to work out ten various optimum frames as in Table 6.3.

**Table 6. 3: Optimum design results of three-storey, two-bay frame with semi-rigid connection (SCS).**

| Three-Storey, two-bay frame | | |
|---|---|---|
| | Semi-rigid Connection (SCS) | |
| **Frame Analysis** | **Optimum Weight** | **Max Improvisation** |
| **1** | 6852 | 1452 |
| **2** | 6492 | 2005 |
| **3** | 6744 | 1490 |
| **4** | 6504 | 1558 |
| **5** | 6432 | 1546 |
| **6** | **6300** | **1547** |
| **7** | 6336 | 1513 |
| **8** | 6504 | 1558 |
| **9** | 6756 | 1674 |
| **10** | 6432 | 1249 |
| **Min ( Ib )** | **6300** | |
| **Average ( Ib )** | **6535** | |
| **Standard Deviation ( Ib )** | **186** | |

According to the results obtained from ten independent runs, semi-rigid connection in both FCS and SCS has the same optimum weight which is 6300 Ib but the iteration is completely different from each other. However, The Full Catalog

Section analysis showed that the minimum weight is obtained at the 2366-th iterations, while the optimum weight in the Selected Catalog Section analysis achieved at 1547-th iterations. This is because the SCS has flexibility in choosing beams and columns while in FCS, there is some difficulty in choosing section as beam and column sections which are not already identified.

Figure 6.4 displays the optimum design history; the weight is unchanged from 1547-th iterations to the maximum numbers 2500-th iterations.

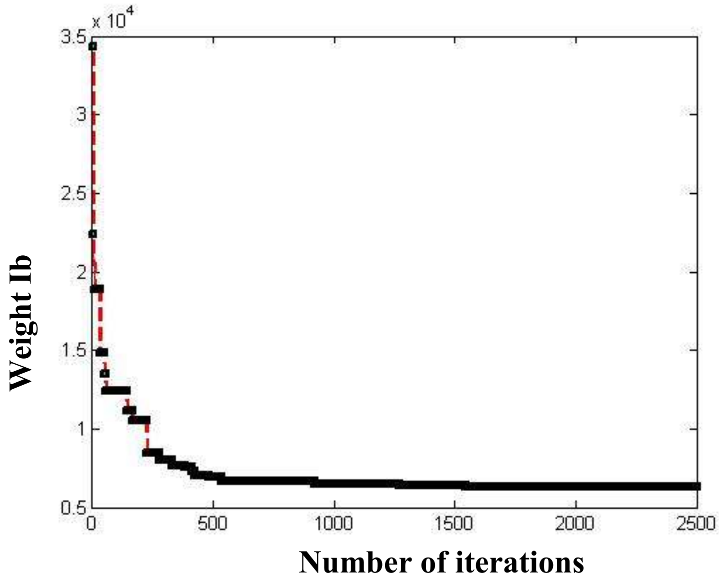


**Figure 6. 4: Optimum design history of three-storey, two-bay semi-rigid frame (SCS).**

### 6.2.3 Comparison between linear and non-linear analysis.

The design algorithm presented is used to design three-storey, two-bay steel frames with semi-rigid connections taking into consideration the linear and nonlinear (P–Δ) effect as in Figure 6.1.

Selected Catalog Section (SCS) are being used because the previous results prove that it can reach the appropriate solution more rapidly and gives the optimum weight. Table 6.4 clarified the best optimum design achieved.

**Table 6. 4: Optimum designs for a three-storey, two-bay steel frame with linear and non-linear analysis.**

| Group | Member type | Harmony search optimization algorithm | |
|---|---|---|---|
| | | Extend end plate without column stiffeners | |
| | | Linear Analysis | Non-linear Analysis |
| | | Selected Catalog Section (SCS) | |
| 1 | Column | W14X53 | W12X35 |
| 2 | Column | W12X26 | W12X26 |
| 3 | Column | W8X21 | W8X24 |
| 4 | Column | W14X43 | W14X43 |
| 5 | Column | W14X43 | W12X30 |
| 6 | Column | W10X22 | W10X22 |
| 7 | Beam | W14X26 | W16X26 |
| Total weight (Ib) | | 6816 | 6300 |
| Top storey sway (in) Allowable = 1.44 in | | 0.85 | 0.92 |
| Saving weight | | 7.57 % | |

The results prove that the optimum design with non-linear analyses is better than that of the linear analysis as in Figure 6.5; approximately 7.57% reduction in weight was obtained.

On the other hand, if the overall gravity loading is not that large compared to lateral loading, geometric nonlinearity in the frame design yields lighter frames compared to linear frames. Even though, the solution with linear analysis requires one hour and a half, which the non-linear analysis needs more than one hour, almost three hours.

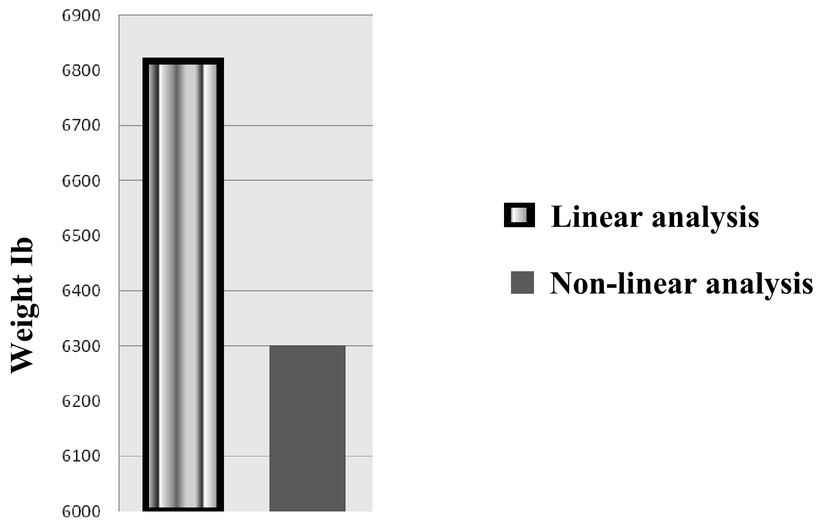


**Figure 6. 5: Optimum design of three-storey, two-bay using linear and non-linear analysis).**

### 6.2.4 Comparison of optimization results between HS and GA.

The three-storey, two-bay steel frame was also investigated by Kameshki and Saka (2003) [27]. Table 6.5 compares the optimum design results produced by GAs with those obtained using HS algorithm.

**Table 6. 5: Comparison of optimization results between HS and GA.**

| | | Non-linear frame analysis | | | | |
|---|---|---|---|---|---|---|
| **Group** | **Member type** | GA (Saka , 2003) | | HS | | |
| | | Rigid | Extend end plate without column stiffeners | Rigid | Extend end plate without column stiffeners | |
| | | BS5950 | BS5950 | FCS | FCS | SCS |
| 1 | Column | W24X55 | W18X36 | W21X48 | W18X40 | W12X35 |
| 2 | Column | W16X31 | W14X26 | W12X26 | W12X26 | W12X26 |
| 3 | Column | W12X40 | W8X18 | W10X22 | W8X21 | W8X24 |
| 4 | Column | W18X35 | W24X68 | W16X40 | W16X40 | W14X43 |
| 5 | Column | W18X35 | W24X68 | W12X30 | W12X30 | W12X30 |
| 6 | Column | W12X35 | W18X35 | W10X22 | W8X21 | W10X22 |
| 7 | Beam | W16X26 | W16X26 | W16X26 | W14X26 | W16X26 |
| Total weight (Ib) | | 7404 | 7092 | 6528 | 6300 | 6300 |
| Top storey sway (in) Allowable = 1.44 in | | 0.64 | 0.61 | 0.63 | 0.93 | 0.92 |

Based on the results obtained from Table 6.5, the HS yielded 11.8 % lighter frames in comparison with GAs in case of rigid frame. Moreover, it is observed from Table 6.5 that HS yielded 11.2 % lighter frames compared with GAs in case of semi-rigid frame.

The sway values at the top storey are lower than their limitation value according to AISC-LRFD in case of GAs and HS. Moreover, the top storey sway is increased in case of semi-rigid frame due to reduction in stiffness.

Figure 6.6 compares the design results that are produced by GAs with the results obtained by HS algorithm. The result indicates that the semi-rigid frame is lighter than that of the rigid frame connection. In addition, there is no different in optimum weight when using FCS and SCS, although the sections are completely different for both cases.

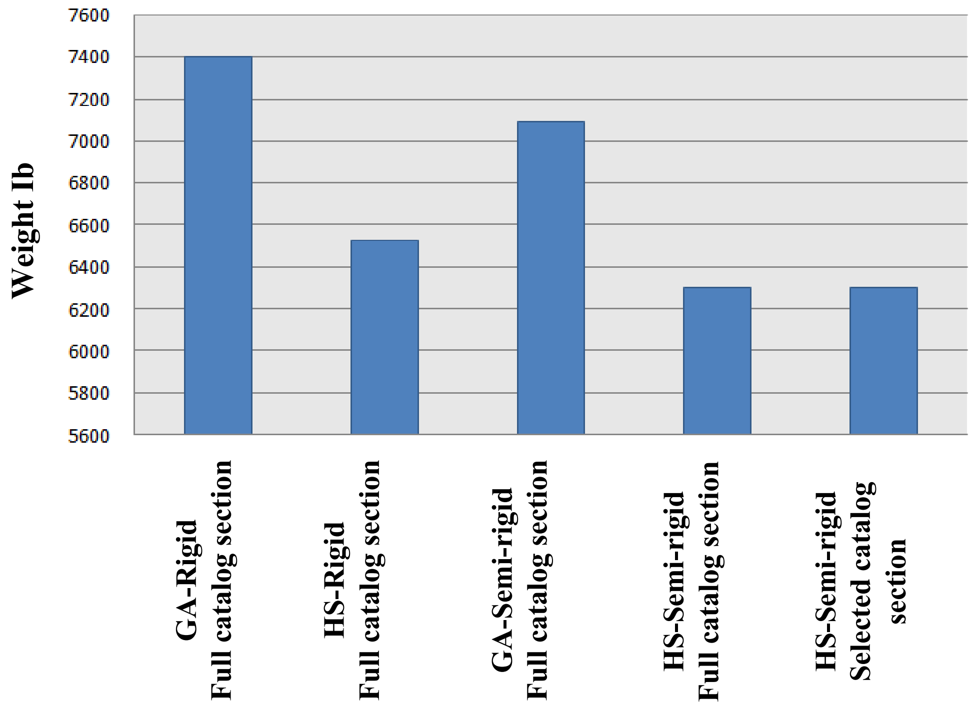


**Figure 6. 6: Comparison of optimum designs for three-storey, two-bay frame.**

## 6.3 Optimization of ten-storey, one-bay steel framed structure.

A ten-storey, one-bay steel frame structure optimization using HS search algorithm is presented in this chapter using various assumptions, in order to compare the results of the HS algorithm with results of an identical structure being optimized using the Genetic Algorithm Optimization Technique (GA). The structure has been analyzed assuming rigid beam-to-column connections and then another analysis has been carried out assuming semi-rigid connections using the FCS and SCS. The analysis has been run twice for each of the previously mentioned assumptions, once considering a linear behavior and then assuming a non-linear behavior. Finally, the results of all analysis have been compared to those in its GA counterpart. The structure being optimized is shown in Figure 6.7.

The design constant parameters which are going to be used the same as in the previous model, except these parameters.

- Allowable top storey sway (H/300) = 4.92 in.
- Allowable interstorey sway (h/300) = 0.48 in.
- Allowable deflection for service dead and live load (L/240) = 1.5 in.

For the HS algorithm not to be stuck in local optimum solutions, and to encourage the HS algorithm to search the global spectrum of solutions, a set of very-carefully-selected parameters have been established by various trials, and recommendations found in the literature review.

The obtained values of the HS tuning parameter are as follows:

- Harmony memory size (*HMS*) =20.
- Harmony memory consideration rate *(HMCR)* = 0.9.
- Pitch adjusting rate *(PAR)* = 0.45.
- Neighbouring index used in the pitch-adjustment ± 2.

Termination criterions obtained after different optimum designs trials.

- First termination = 1000-th iterations.
- Second termination = 5000-th iterations.

Ten independent runs are made to minimize the weight of the steel frames with rigid and semi-rigid connections.

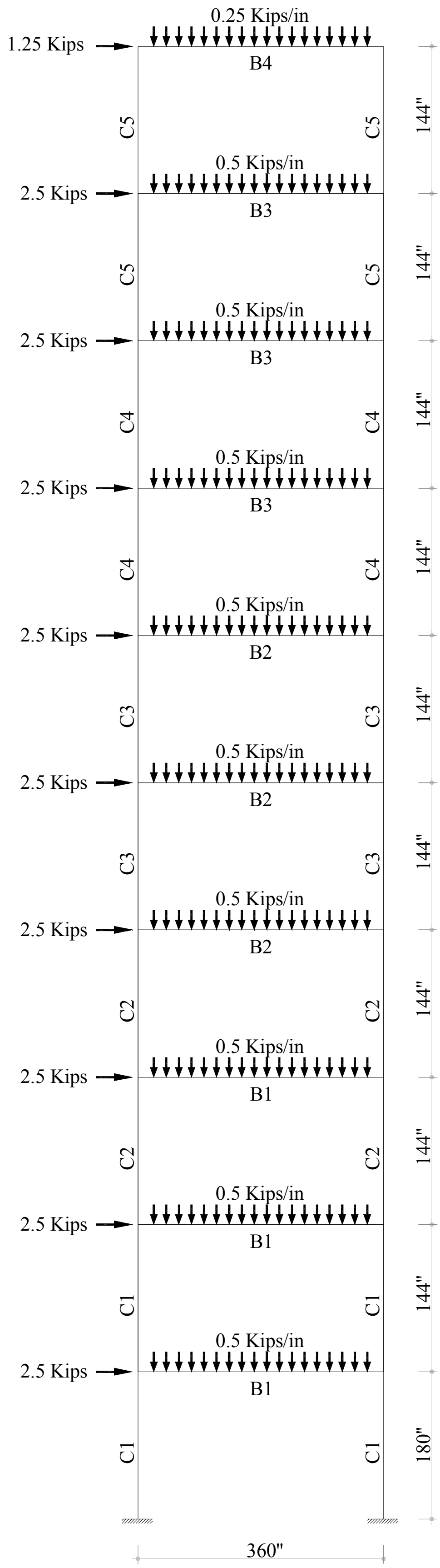


**Figure 6. 7: Ten-storey, one-bay frame.**

### 6.3.1 Optimization by analyzing rigid frame structure.

Ten independent optimum frame designs are achieved using Full Catalog Section (FCS) selection as in Appendix-B; The ten-storey, one-bay steel frame analyzed by linear analysis. The result is presented in Table 6.6.

**Table 6. 6: Optimum design results of ten-storey, one-bay frame with rigid connection (FCS).**

| Ten-Storey, one-bay frame | | |
|---|---|---|
| | **Rigid Connection ( FCS )** | |
| **Frame Analysis** | **Optimum Weight** | **Max Improvisation** |
| **1** | 48972 | 4919 |
| **2** | 48984 | 4244 |
| **3** | 51600 | 2071 |
| **4** | 50106 | 3674 |
| **5** | 48984 | 4244 |
| **6** | 49230 | 4908 |
| **7** | 49242 | 2468 |
| **8** | 49086 | 3105 |
| **9** | 52368 | 2982 |
| **10** | **48828** | **4122** |
| **Min ( lb )** | **48828** | |
| **Average ( lb )** | **49651** | |
| **Standard Deviation ( lb )** | **1056** | |

Table 6.6 represented the result of the optimization process. It is noticed that HS develops the optimum design weight at 4122-th iterations and it remains unchanged until the maximum number of iteration reaches 5000-th. The average weight and a standard deviation are 49,651 lb, 1056 lb respectively. The maximum sway obtained at the optimum design is 0.91 in which smaller than the allowable limit by AISC-LRFD 4.92 in.

The optimum design history is shown in Figure 6.8. The Figure shows that the optimization process is converged rapidly in the first 1000-th iterations, after that in minimum curve remains almost unchanged until 5000-th iterations obtained.

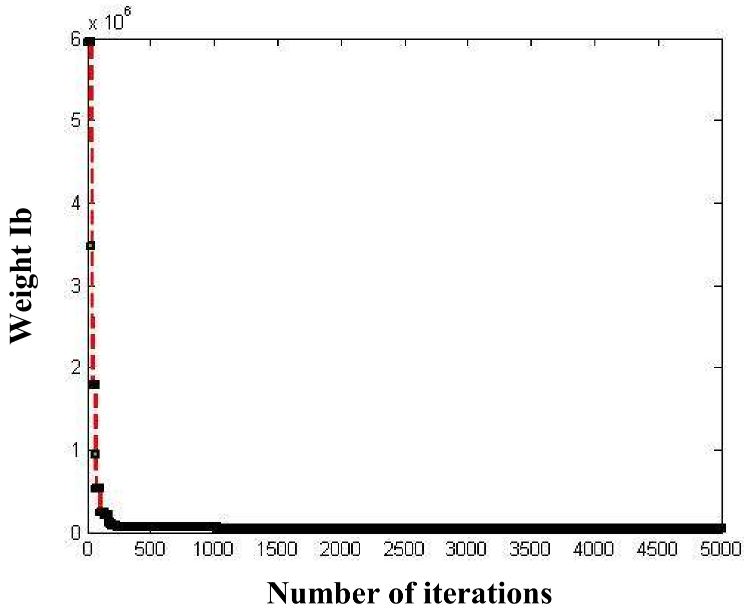


**Figure 6. 8: Optimum design history of ten-storey, one-bay rigid frame (FCS).**

### 6.3.2 Optimization by analyzing the connection semi-rigid frame.

This case study is worked out using semi-rigid connections, using also Frye–Morris polynomial model to present the connection behaviour.

#### 6.3.2.1 Optimization results using FCS.

A ten-storey, one-bay steel frame has the same geometric and loading is shown in Figure 6.7. The results of ten independent runs are achieved as in Table 6.7.

**Table 6. 7: Optimum design results of ten-storey, one-bay frame with semi-rigid connection (FCS).**

| Ten-Storey, one-bay frame | | |
|---|---|---|
| Semi-rigid Connection ( Full Catalog Section ) | | |
| **Frame Analysis** | **Optimum Weight** | **Max Improvisation** |
| **1** | 50316 | 3323 |
| **2** | 52428 | 4395 |
| **3** | 50316 | 3323 |
| **4** | 49068 | 4994 |
| **5** | 49134 | 3627 |
| **6** | 49248 | 2954 |
| **7** | **48744** | **3492** |
| **8** | 51792 | 2948 |
| **9** | 49734 | 4690 |
| **10** | 51810 | 4908 |
| **Min ( Ib )** | **48744** | |
| **Average ( Ib )** | **50259** | |
| **Standard Deviation ( Ib)** | **1323** | |

The optimum weight in case of the structure with analysis as semi-rigid connections is found to be 48,744 Ib. This value is less than the optimum weight using the rigid one 48,828 Ib, even though both of them has the same catalog (FCS). The results observed that the standard deviation is 1056 Ib in case of rigid frame, which is less than 1323 Ib in case of semi-rigid frame, which can refers due to lack of stiffness.

Figure 6.9 displays the optimum design history. It is observed from the Figure; the frame weight at 1000-th iterations which equals 49,392 Ib and becomes 48,744 Ib after 5000-th iterations this means that the weight decreases only about 1.31 % for 4000-th iterations.

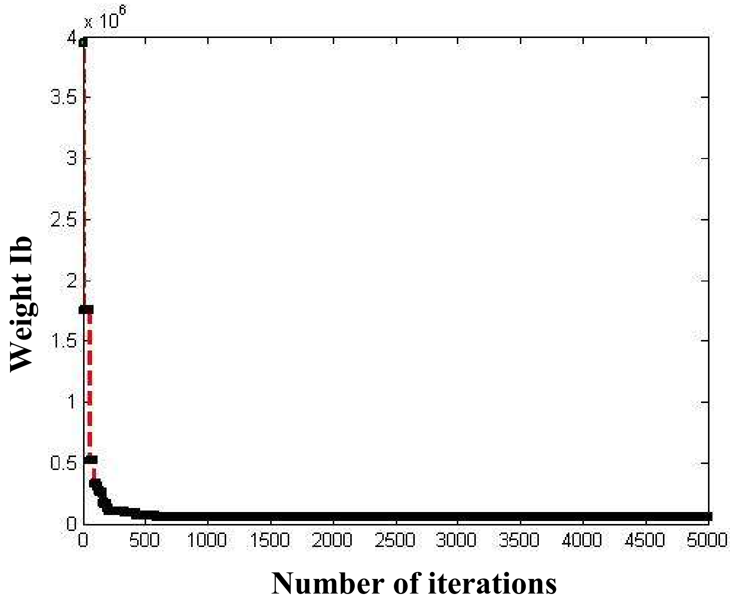


**Figure 6. 9: Optimum design history of ten-storey, one-bay semi-rigid frame (FCS).**

### 6.3.2.2 Optimization results using SCS.

Table 6.8 represented the results of ten independent runs. It is observed that HS develops the optimum design weight 47,832 Ib at 4122-th iterations and it remains unchanged until the maximum number of iteration reaches 5000-th. Moreover, the standard deviation increases about 38 % in comparable with optimum design using FCS.

**Table 6. 8: Optimum design results of ten-storey, one-bay frame with semi-rigid connection (SCS).**

| Ten-Storey, one-bay frame | | |
|---|---|---|
| | **Semi-rigid Connection ( SCS )** | |
| **Frame Analysis** | **Optimum Weight** | **Max Improvisation** |
| **1** | 51996 | 2670 |
| **2** | 51996 | 2670 |
| **3** | 50484 | 2933 |
| **4** | 54000 | 3074 |
| **5** | 54042 | 4155 |
| **6** | **47832** | **4050** |
| **7** | 52206 | 4190 |
| **8** | 49956 | 3186 |
| **9** | 50082 | 4250 |
| **10** | 48216 | 3916 |
| **Min ( Ib )** | **47832** | |
| **Average ( Ib )** | **51081** | |
| **Standard Deviation ( Ib )** | **2150** | |

The optimum design history is shown in Figure 6.10. The optimum weight at 1000-th iterations equal 48,516 Ib and become 47,832 Ib after 5000-th iterations this means that the weight decrease only about 1.40 % for the last 4000-th iterations.

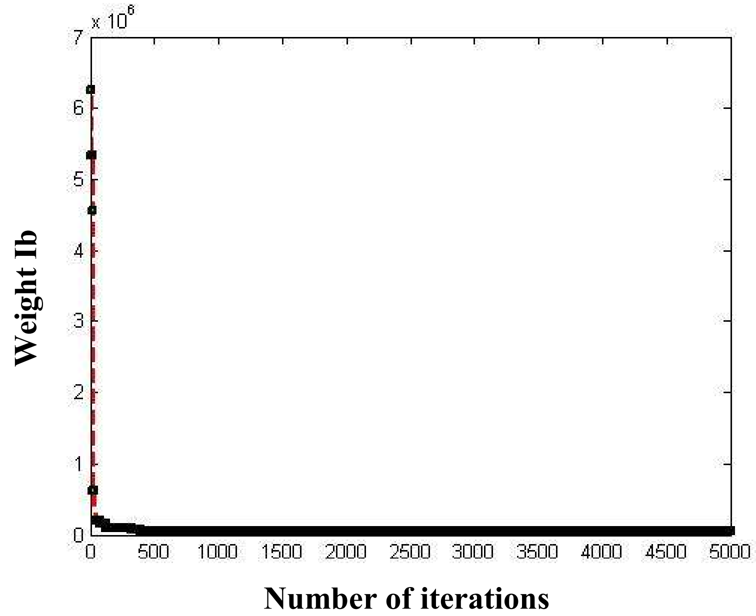


**Figure 6. 10: Optimum design history of ten-storey, one-bay semi-rigid frame (SCS).**

### 6.3.3 Comparison between linear and non-linear analysis.

The design algorithm presented is used to design ten-storey, one-bay steel frames with semi-rigid connections taking into consideration the linear and nonlinear (P–Δ) effect as in Figure 6.7.

Selected Catalog Section (SCS) are being used because the previous results prove that it can reach the appropriate solution more rapidly and gives the optimum weight. Table 6.9 tabulates the optimum design sections achieved.

**Table 6. 9: Comparison between linear and non-linear analysis.**

| Group | Member type | Harmony search optimization algorithm | |
|---|---|---|---|
| | | Extend end plate without column stiffeners | |
| | | Linear Analysis | Non-linear Analysis |
| | | Selected Catalog Section (SCS) | |
| 1 | Column | W27X146 | W27X146 |
| 2 | Column | W21X122 | W24X131 |
| 3 | Column | W21X101 | W21X101 |
| 4 | Column | W18X76 | W16X89 |
| 5 | Column | W14X82 | W14X82 |
| 6 | Beam | W24X68 | W24X68 |
| 7 | Beam | W24X68 | W24X76 |
| 8 | Beam | W27X84 | W24X94 |
| 9 | Beam | W21X62 | W21X62 |
| Total weight (Ib) | | 47,832 | 50508 |
| Top storey sway (in) Allowable = 4.92 in | | 1.43 | 1.55 |
| Saving weight | | 5.3 % | |

The results prove that the solution with linear analyses is better than that of the non-linear analysis as in Figure 6.11, approximately 5.3% in weight. Moreover, the result indicates that when the overall gravity loading is much larger compared to lateral loading and is dominant in the design of the frame, linear semi-rigid frames are lighter than non-linear semi-rigid frames. So not surprisingly that the solution with linear analysis requires three hour, which the non-linear analysis needs more than three hour, almost nine hours; the properties of the used computer are:

- Computer type: Dell Inspiron.
- Processor: Pentium (R) dual-core CPU.
- Installed memory: 4.00 GB.
- System type: 64 bit operating system.

On the other hand, the linear analyses require two hours with processor core I3. So the results depend on computer type and specification.

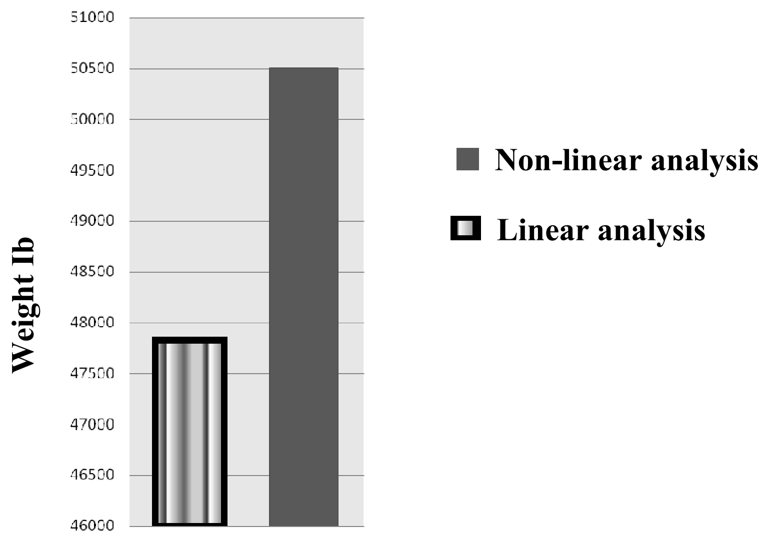


**Figure 6. 11: Optimum design of ten-storey, one-bay using linear and non-linear analysis.**

### 6.3.4 Comparison of optimum results between HS and GA.

The ten-storey, one-bay steel frame was also investigated by Kameshki and Saka (2003) [27]. Table 6.10 compares the optimum design results produced by GAs with those obtained using HS algorithm.

Based on the results obtained from Table 6.10, the HS yielded 5.18 % lighter frames in comparison with GAs in case of rigid frame. In addition, it is observed from Table that HS yielded 7.76 % lighter frames compared with GAs in case of semi-rigid frame.

The sway values at the top storey are lower than their limitation value according to AISC-LRFD in case of GAs and HS. Moreover, the top storey sway is increased in case of semi-rigid frame due to reduction in stiffness.

**Table 6. 10: Comparison of optimum results between HS and GA.**

| Group | Member type | Linear frame analysis | | | | |
|---|---|---|---|---|---|---|
| | | GA (Saka , 2003) | | HS | | |
| | | Rigid | Extend end plate without column stiffeners | Rigid | Extend end plate without column stiffeners | |
| | | BS5950 | BS5950 | FCS | FCS | SCS |
| 1 | Column | W36X135 | W36X160 | W36X150 | W24X162 | W27X146 |
| 2 | Column | W33X141 | W36X135 | W30X132 | W24X131 | W21X122 |
| 3 | Column | W30X108 | W36X135 | W27X114 | W21X101 | W21X101 |
| 4 | Column | W27X102 | W33X118 | W24X84 | W14X82 | W18X76 |
| 5 | Column | W14X90 | W30X108 | W18X76 | W14X68 | W14X82 |
| 6 | Beam | W24X68 | W24X68 | W24X76 | W24X68 | W24X68 |
| 7 | Beam | W24X68 | W24X68 | W24X76 | W24X68 | W24X68 |
| 8 | Beam | W27X84 | W24X68 | W24X68 | W27X84 | W27X84 |
| 9 | Beam | W30X108 | W18X35 | W21X48 | W21X62 | W21X62 |
| Total weight (Ib) | | 51,498 | 51,858 | 48,828 | 48,744 | 47,832 |
| Top storey sway (in) Allowable = 4.92 in | | 0.93 | 1.21 | 0.91 | 1.45 | 1.43 |

Figure 6.6 compares the design results that are produced by GAs with the results obtained by HS algorithm. The result indicates that the semi-rigid frame connections is lighter than that of the rigid frame connection. In addition, Selected Catalog Section (SCS) are lighter than Full Catalog Section (FCS) about 1.87 per cent.

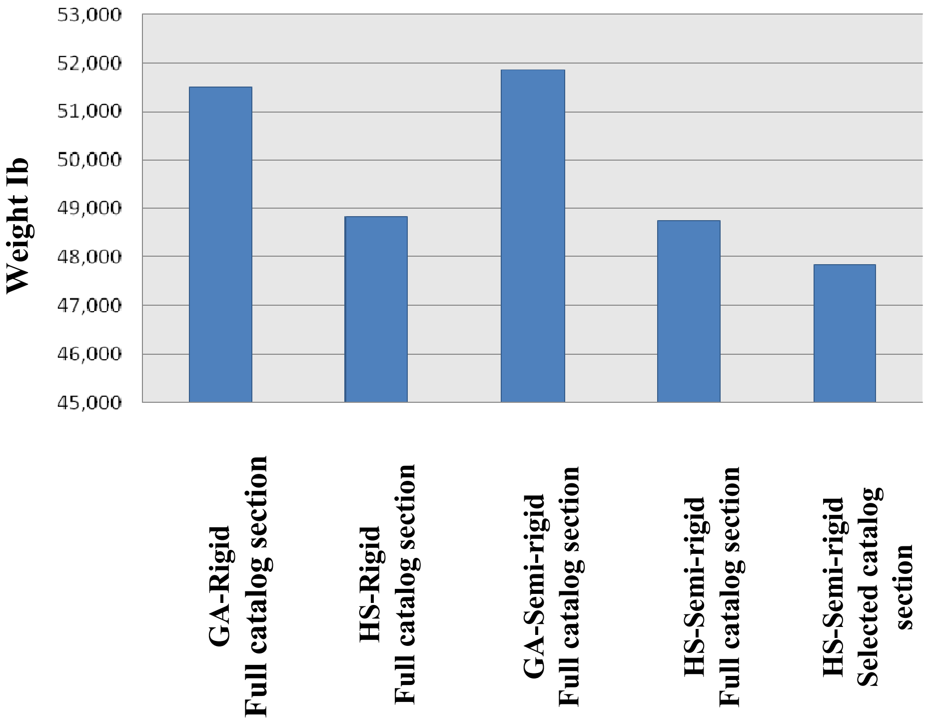


**Figure 6. 12: Comparison of optimum designs for ten-storey, one-bay frame.**

# CHAPTER 7 : CONCLUSION AND FUTURE RESEARCH

## 7.1 Introduction.

The aim of this research is to develop a computer design model which obtains the optimum frame weight by selecting a standard set of steel sections and satisfy strength constraints of AISC-LRFD specification, displacement constraints, deflection and also size constraint for beam-columns were imposed on frames.

The recently developed HS meta-heuristic optimization algorithm was conceptualized using the musical process of searching for a perfect state of harmony. Compared to gradient-based mathematical optimization algorithms, the HS algorithm imposes fewer mathematical requirements and does not require initial value settings of the decision variables. As the HS algorithm uses stochastic random searches, derivative information is also unnecessary. Furthermore, the HS algorithm generates a new vector, after considering all of the existing vectors based on the harmony memory considering rate (*HMCR*) and the pitch adjusting rate (*PAR*), whereas the GA only consider the two parent vectors. These features increase the flexibility of the HS algorithm and produce better solutions.

## 7.2 Conclusion.

Optimum design of semi-rigid steel frame structures using harmony search algorithm has been achieved in this study. The conclusions can be summarized as follows:

1. HS algorithm developed 5.18 –11.8 % lighter frames in the case of rigid connections compared to ones produced by GAs.
2. HS algorithm developed 7.76 –11.2 % lighter frames in the case of semi-rigid connections compared to ones produced by GAs.
3. Optimization using Selected Catalog Section (SCS) result in lighter frame sections than using Full Catalog Section (FCS) about 1.87%.
4. HS converges to optimum designs before the maximum number of frame analyses is executed in almost all designs.
5. The optimum design weight decreases gradually after 1000-th iterations only about 1.3-1.4% to reach the maximum number of iterations.
6. The designs with semi-rigid connection resulted in lighter frames than the ones with rigid connections. In addition, the total costs of the flexible connected frames are less than the rigidly connected frames.
7. The result using harmony search algorithm prove that is powerful and effective tools, because HS generates a new design considering all

existing designs, while GA generates a new design from a couple of chosen parents by exchanging the artificial genes. On the other hand, HS takes into account each design variable independently but GA considers design variables depending upon building block theory.

## 7.3 Future research.

There are many ways to develop new algorithms, and from the metaheuristic point of view, the most heuristic way is probably to develop new algorithms by hybridization. That is to say, new algorithms are often based on the right combination of the existing metaheuristic algorithms. For example, combining a trajectory type simulated annealing with multiple agents; the parallel simulated annealing optimization (PSO) can be developed. In the context of HS algorithms, the combination of HS with PSO. As in the case of any efficient metaheuristic algorithms, the most difficult thing is probably to find the right or optimal balance between diversity and intensity of the found solutions; here the most challenging task in developing new hybrid algorithms is probably to find the right combination of which feature/components of existing algorithms.

A future extension adaptive harmony search algorithm can be employed with confidence in the optimum design of real size steel skeletal structures. In this technique, the harmony search parameters are dynamically adjusted by the algorithm itself taking into account varying features of the design problem under consideration. The algorithm itself automatically changes the values of harmony considering rate *(HMCR)* and pitch adjustment rate *(PAR)* depending on the experience obtained through the design process. Hence, varying features of a design space are automatically accounted by the algorithm for establishing a tradeoff between explorative and exploitative search for the most successful optimization process. Finally, the adaptive harmony search algorithm eliminates the necessity of carrying out a sensitivity analysis with different values of harmony search parameters whenever a new design problem is to be undertaken. This makes the algorithm more general and applicable to the optimum design of large size real-world steel structures.

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# APPENDIX-A

## 1- Input file for nonlinear analysis of semi-rigid frame (three-storey, two-bay)

```
/PREP7
/NOPR
/PMETH,OFF,0
KEYW,PR_SET,1
KEYW,PR_STRUC,1
KEYW,PR_THERM,0
KEYW,PR_FLUID,0
KEYW,PR_ELMAG,0
KEYW,MAGNOD,0
KEYW,MAGEDG,0
KEYW,MAGHFE,0
KEYW,MAGELC,0
KEYW,PR_MULTI,0
KEYW,PR_CFD,0
/GO
/PREP7
!*** ELEMENT TYPES ***
!*
ET,1,BEAM3
ET,2,COMBIN39
!*
KEYOPT,1,6,1
KEYOPT,1,9,0
KEYOPT,1,10,0
!*
KEYOPT,2,1,0
KEYOPT,2,2,0
KEYOPT,2,3,6
KEYOPT,2,4,0
KEYOPT,2,6,0
!*
!*** REAL CONSTANTS ***
!*** COLUMN ***
R,1,10.3,285,12.5,0,0,0
R,2,7.65,204,12.2,0,0,0
R,3,7.08,82.7,7.93,0,0,0
R,4,12.6,428,13.7,0,0,0
R,5,8.79,238,12.3,0,0,0
R,6,6.49,118,10.2,0,0,0
R,7,7.68,301,15.7,0,0,0
!*
!*** NONLINEAR SPRING ***
R,8,0,0,0.0005,378,0.005,2900
RMORE,0.01,4200,0.015,4960,0.02,5500
```

```
!*** MATERIAL BEHAVIOR ***
MPTEMP,,,,,,,,
MPTEMP,1,0
MPDATA,EX,1,,30000
MPDATA,PRXY,1,,0.30
!*
!*** KEYPOINT ***
!*
K,1,0,0,0,
K,2,240,0,0,
K,3,480,0,0,
K,4,0,144,0,
K,5,240,144,0,
K,6,480,144,0,
K,7,0,288,0,
K,8,240,288,0,
K,9,480,288,0,
K,10,0,432,0,
K,11,240,432,0,
K,12,480,432,0,
!*
!*** SPRING KEYPOINT ***
K,13,0,144,0,
K,14,240,144,0,
K,15,240,144,0,
K,16,480,144,0,
K,17,0,288,0,
K,18,240,288,0,
K,19,240,288,0,
K,20,480,288,0,
K,21,0,432,0,
K,22,240,432,0,
K,23,240,432,0,
K,24,480,432,0,
!*
!*** ELEMENT LINE ***
LSTR,       1,       4
LSTR,       3,       6
LSTR,       4,       7
LSTR,       6,       9
LSTR,       7,      10
LSTR,       9,      12
LSTR,       2,       5
LSTR,       5,       8
LSTR,       8,      11
!*
LSTR,       13,      14
LSTR,       15,      16
LSTR,       17,      18
```

```
LSTR,       19,       20
LSTR,       21,       22
LSTR,       23,       24
!*
!*** MESHING LINE ***
FLST,5,2,4,ORDE,2
FITEM,5,1
FITEM,5,-2
CM,_Y,LINE
LSEL, , , ,P51X
CM,_Y1,LINE
CMSEL,S,_Y
CMSEL,S,_Y1
LATT,1,1,1, , , ,
CMSEL,S,_Y
CMDELE,_Y
CMDELE,_Y1
FLST,5,2,4,ORDE,2
FITEM,5,3
FITEM,5,-4
CM,_Y,LINE
LSEL, , , ,P51X
CM,_Y1,LINE
CMSEL,S,_Y
CMSEL,S,_Y1
LATT,1,2,1, , , ,
CMSEL,S,_Y
CMDELE,_Y
CMDELE,_Y1
FLST,5,2,4,ORDE,2
FITEM,5,5
FITEM,5,-6
CM,_Y,LINE
LSEL, , , ,P51X
CM,_Y1,LINE
CMSEL,S,_Y
CMSEL,S,_Y1
LATT,1,3,1, , , ,
CMSEL,S,_Y
CMDELE,_Y
CMDELE,_Y1
CM,_Y,LINE
LSEL, , , ,       7
CM,_Y1,LINE
CMSEL,S,_Y
CMSEL,S,_Y1
LATT,1,4,1, , , ,
CMSEL,S,_Y
CMDELE,_Y
```

```
CMDELE,_Y1
CM,_Y,LINE
LSEL, , , ,       8
CM,_Y1,LINE
CMSEL,S,_Y
CMSEL,S,_Y1
LATT,1,5,1, , , ,
CMSEL,S,_Y
CMDELE,_Y
CMDELE,_Y1
CM,_Y,LINE
LSEL, , , ,       9
CM,_Y1,LINE
CMSEL,S,_Y
CMSEL,S,_Y1
LATT,1,6,1, , , ,
CMSEL,S,_Y
CMDELE,_Y
CMDELE,_Y1
FLST,5,6,4,ORDE,2
FITEM,5,10
FITEM,5,-15
CM,_Y,LINE
LSEL, , , ,P51X
CM,_Y1,LINE
CMSEL,S,_Y
CMSEL,S,_Y1
LATT,1,7,1, , , ,
CMSEL,S,_Y
CMDELE,_Y
CMDELE,_Y1
LESIZE,ALL, , ,10, ,1, , ,1,
FLST,2,15,4,ORDE,2
FITEM,2,1
FITEM,2,-15
LMESH,P51X
!*
!*Spring define*!
TYPE,   2
MAT,       1
REAL,       8
ESYS,       0
SECNUM,
TSHAP,LINE
FLST,2,2,1
FITEM,2,2
FITEM,2,94
E,P51X
FLST,2,2,1
```

```
FITEM,2,64
FITEM,2,95
E,P51X
FLST,2,2,1
FITEM,2,64
FITEM,2,105
E,P51X
FLST,2,2,1
FITEM,2,13
FITEM,2,106
E,P51X
FLST,2,2,1
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FITEM,2,116
E,P51X
FLST,2,2,1
FITEM,2,74
FITEM,2,117
E,P51X
FLST,2,2,1
FITEM,2,74
FITEM,2,127
E,P51X
FLST,2,2,1
FITEM,2,33
FITEM,2,128
E,P51X
FLST,2,2,1
FITEM,2,43
FITEM,2,138
E,P51X
FLST,2,2,1
FITEM,2,84
FITEM,2,139
E,P51X
FLST,2,2,1
FITEM,2,84
FITEM,2,149
E,P51X
FLST,2,2,1
FITEM,2,53
FITEM,2,150
E,P51X
!*
CPINTF,UX,0.0001,
CPINTF,UY,0.0001,
FINISH
/SOL
!*
```

```
ANTYPE,0
NLGEOM,1
NSUBST,20,100,1
LNSRCH,1
AUTOTS,ON
NROPT,FULL,,On
NEQIT,25
OUTRES,ALL,LAST
!*
FLST,2,3,3,ORDE,2
FITEM,2,1
FITEM,2,-3
/GO
DK,P51X, , , ,0,ALL, , , , , ,
FLST,2,2,3,ORDE,2
FITEM,2,4
FITEM,2,7
/GO
FK,P51X,FX,8
FLST,2,1,3,ORDE,1
FITEM,2,10
/GO
FK,P51X,FX,4
FLST,2,40,2,ORDE,2
FITEM,2,91
FITEM,2,-130
SFBEAM,P51X,1,PRES,0.22,0.22, , , , ,
FLST,2,20,2,ORDE,2
FITEM,2,131
FITEM,2,-150
SFBEAM,P51X,1,PRES,0.17,0.17, , , , ,
!*
SAVE
SOLVE
/POST1
!*** ELEMENT PROPERTIS ***
!*
AVPRIN,0, ,
ETABLE,UX,U,X
VPRIN,0, ,
ETABLE,UY,U,Y
AVPRIN,0, ,
ETABLE,PU,SMISC, 1
AVPRIN,0, ,
ETABLE,MI,SMISC, 6
AVPRIN,0, ,
ETABLE,MJ,SMISC, 12
PRETAB,UX,UY,PU,MI,MJ
```

## 2- Input file for linear Analysis of semi-rigid frame (ten-storey, one-bay).

```
/PREP7
/NOPR
/PMETH,OFF,0
KEYW,PR_SET,1
KEYW,PR_STRUC,1
KEYW,PR_THERM,0
KEYW,PR_FLUID,0
KEYW,PR_ELMAG,0
KEYW,MAGNOD,0
KEYW,MAGEDG,0
KEYW,MAGHFE,0
KEYW,MAGELC,0
KEYW,PR_MULTI,0
KEYW,PR_CFD,0
/GO
!*** ELEMENT TYPES ***
ET,1,BEAM3
ET,2,COMBIN39
!*
KEYOPT,1,6,1
KEYOPT,1,9,0
KEYOPT,1,10,0
!*
KEYOPT,2,1,0
KEYOPT,2,2,0
KEYOPT,2,3,6
KEYOPT,2,4,0
KEYOPT,2,6,0
!*
!*** REAL CONSTANTS ***
!*** COLUMN ***
R,1,43.1,5660,27.4,0,0,0
R,2,35.9,2960,21.7,0,0,0
R,3,29.8,2420,21.4,0,0,0
R,4,22.3,1330,18.2,0,0,0
R,5,24,881,14.3,0,0,0
R,6,20.1,1830,23.7,0,0,0
R,7,20.1,1830,23.7,0,0,0
R,8,24.8,2850,26.7,0,0,0
R,9,18.3,1330,21,0,0,0
!*
!*** NONLINEAR SPRING ***
R,10,0,0,0.0005,1112,0.005,8510
RMORE,0.01,12350,0.015,14605,0.02,16210
R,11,0,0,0.0005,1112,0.005,8510
RMORE,0.01,12350,0.015,14605,0.02,16210
R,12,0,0,0.0005,1420,0.005,10730
```

```
RMORE,0.01,15600,0.015,18400,0.02,20418
R,13,0,0,0.0005,885,0.005,6780
RMORE,0.01,9825,0.015,11618,0.02,12900
!*
!*** MATERIAL BEHAVIOR ***
MPTEMP,,,,,,,,
MPTEMP,1,0
MPDATA,EX,1,,30000
MPDATA,PRXY,1,,0.30
!*
!*** KEYPOINT ***
K,1,0,0,0,
K,3,0,144,0,
K,5,0,288,0,
K,7,0,432,0,
K,9,0,576,0,
K,11,0,720,0,
K,13,0,864,0,
K,15,0,1008,0,
K,17,0,1152,0,
K,19,0,1296,0,
K,21,0,1440,0,
!*
K,2,360,0,0,
K,4,360,144,0,
K,6,360,288,0,
K,8,360,432,0,
K,10,360,576,0,
K,12,360,720,0,
K,14,360,864,0,
K,16,360,1008,0,
K,18,360,1152,0,
K,20,360,1296,0,
K,22,360,1440,0,
!*
!*** SPRING KEYPOINT ***
K,23,0,144,0,
K,25,0,288,0,
K,27,0,432,0,
K,29,0,576,0,
K,31,0,720,0,
K,33,0,864,0,
K,35,0,1008,0,
K,37,0,1152,0,
K,39,0,1296,0,
K,41,0,1440,0,
!*
K,24,360,144,0,
K,26,360,288,0,
```

```
K,28,360,432,0,
K,30,360,576,0,
K,32,360,720,0,
K,34,360,864,0,
K,36,360,1008,0,
K,38,360,1152,0,
K,40,360,1296,0,
K,42,360,1440,0,
!*
!*** ELEMENT LINE ***
LSTR,       1,       3
LSTR,       2,       4
LSTR,       3,       5
LSTR,       4,       6
LSTR,       5,       7
LSTR,       6,       8
LSTR,       7,       9
LSTR,       8,      10
LSTR,       9,      11
LSTR,      10,      12
LSTR,      11,      13
LSTR,      12,      14
LSTR,      13,      15
LSTR,      14,      16
LSTR,      15,      17
LSTR,      16,      18
LSTR,      17,      19
LSTR,      18,      20
LSTR,      19,      21
LSTR,      20,      22
LSTR,      23,      24
LSTR,      25,      26
LSTR,      27,      28
LSTR,      29,      30
LSTR,      31,      32
LSTR,      33,      34
LSTR,      35,      36
LSTR,      37,      38
LSTR,      39,      40
LSTR,      41,      42
!*
!*** MESHING LINE ***
FLST,5,4,4,ORDE,2
FITEM,5,1
FITEM,5,-4
CM,_Y,LINE
LSEL, , , ,P51X
CM,_Y1,LINE
CMSEL,S,_Y
```

```
CMSEL,S,_Y1
LATT,1,1,1, , , ,
CMSEL,S,_Y
CMDELE,_Y
CMDELE,_Y1
!*
FLST,5,4,4,ORDE,2
FITEM,5,5
FITEM,5,-8
CM,_Y,LINE
LSEL, , , ,P51X
CM,_Y1,LINE
CMSEL,S,_Y
!*
CMSEL,S,_Y1
LATT,1,2,1, , , ,
CMSEL,S,_Y
CMDELE,_Y
CMDELE,_Y1
!*
FLST,5,4,4,ORDE,2
FITEM,5,9
FITEM,5,-12
CM,_Y,LINE
LSEL, , , ,P51X
CM,_Y1,LINE
CMSEL,S,_Y
!*
CMSEL,S,_Y1
LATT,1,3,1, , , ,
CMSEL,S,_Y
CMDELE,_Y
CMDELE,_Y1
!*
FLST,5,4,4,ORDE,2
FITEM,5,13
FITEM,5,-16
CM,_Y,LINE
LSEL, , , ,P51X
CM,_Y1,LINE
CMSEL,S,_Y
!*
CMSEL,S,_Y1
LATT,1,4,1, , , ,
CMSEL,S,_Y
CMDELE,_Y
CMDELE,_Y1
!*
FLST,5,4,4,ORDE,2
```

```
FITEM,5,17
FITEM,5,-20
CM,_Y,LINE
LSEL, , , ,P51X
CM,_Y1,LINE
CMSEL,S,_Y
!*
CMSEL,S,_Y1
LATT,1,5,1, , , ,
CMSEL,S,_Y
CMDELE,_Y
CMDELE,_Y1
!*
FLST,5,3,4,ORDE,2
FITEM,5,21
FITEM,5,-23
CM,_Y,LINE
LSEL, , , ,P51X
CM,_Y1,LINE
CMSEL,S,_Y
!*
CMSEL,S,_Y1
LATT,1,6,1, , , ,
CMSEL,S,_Y
CMDELE,_Y
CMDELE,_Y1
!*
FLST,5,3,4,ORDE,2
FITEM,5,24
FITEM,5,-26
CM,_Y,LINE
LSEL, , , ,P51X
CM,_Y1,LINE
CMSEL,S,_Y
!*
CMSEL,S,_Y1
LATT,1,7,1, , , ,
CMSEL,S,_Y
CMDELE,_Y
CMDELE,_Y1
!*
FLST,5,3,4,ORDE,2
FITEM,5,27
FITEM,5,-29
CM,_Y,LINE
LSEL, , , ,P51X
CM,_Y1,LINE
CMSEL,S,_Y
!*
```

```
CMSEL,S,_Y1
LATT,1,8,1, , , ,
CMSEL,S,_Y
CMDELE,_Y
CMDELE,_Y1
!*
CM,_Y,LINE
LSEL, , , ,       30
CM,_Y1,LINE
CMSEL,S,_Y
!*
CMSEL,S,_Y1
LATT,1,9,1, , , ,
CMSEL,S,_Y
CMDELE,_Y
CMDELE,_Y1
!*
LESIZE,ALL, , ,10, ,1, , ,1,
FLST,2,30,4,ORDE,2
FITEM,2,1
FITEM,2,-30
LMESH,P51X
!*
TYPE,   2
MAT,       1
REAL,      10
ESYS,       0
SECNUM,
TSHAP,LINE
!*
FLST,2,2,1
FITEM,2,2
FITEM,2,203
E,P51X
FLST,2,2,1
FITEM,2,13
FITEM,2,204
E,P51X
FLST,2,2,1
FITEM,2,23
FITEM,2,214
E,P51X
FLST,2,2,1
FITEM,2,33
FITEM,2,215
E,P51X
FLST,2,2,1
FITEM,2,43
FITEM,2,225
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E,P51X
FLST,2,2,1
FITEM,2,53
FITEM,2,226
E,P51X
!*
TYPE,   2
MAT,       1
REAL,      11
ESYS,       0
SECNUM,
TSHAP,LINE
!*
FLST,2,2,1
FITEM,2,63
FITEM,2,236
E,P51X
FLST,2,2,1
FITEM,2,73
FITEM,2,237
E,P51X
FLST,2,2,1
FITEM,2,83
FITEM,2,247
E,P51X
FLST,2,2,1
FITEM,2,93
FITEM,2,248
E,P51X
FLST,2,2,1
FITEM,2,103
FITEM,2,258
E,P51X
FLST,2,2,1
FITEM,2,113
FITEM,2,259
E,P51X
!*
TYPE,   2
MAT,       1
REAL,      12
ESYS,       0
SECNUM,
TSHAP,LINE
!*
FLST,2,2,1
FITEM,2,123
FITEM,2,269
E,P51X
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FLST,2,2,1
FITEM,2,133
FITEM,2,270
E,P51X
FLST,2,2,1
FITEM,2,143
FITEM,2,280
E,P51X
FLST,2,2,1
FITEM,2,153
FITEM,2,281
E,P51X
FLST,2,2,1
FITEM,2,163
FITEM,2,291
E,P51X
FLST,2,2,1
FITEM,2,173
FITEM,2,292
E,P51X
!*
TYPE,   2
MAT,       1
REAL,      13
ESYS,       0
SECNUM,
TSHAP,LINE
!*
FLST,2,2,1
FITEM,2,183
FITEM,2,302
E,P51X
FLST,2,2,1
FITEM,2,193
FITEM,2,303
E,P51X
!*
CPINTF,UX,0.0001,
CPINTF,UY,0.0001,
!*
ANTYPE,0
!*
FLST,2,2,3,ORDE,2
FITEM,2,1
FITEM,2,-2
DK,P51X, , , ,0,ALL, , , , , ,
!*
FLST,2,1,3,ORDE,1
FITEM,2,3
```

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FLST,2,9,3,ORDE,9
FITEM,2,3
FITEM,2,5
FITEM,2,7
FITEM,2,9
FITEM,2,11
FITEM,2,13
FITEM,2,15
FITEM,2,17
FITEM,2,19
!*
FK,P51X,FX,2.5
FLST,2,1,3,ORDE,1
FITEM,2,21
!*
FK,P51X,FX,1.25
FLST,2,90,2,ORDE,2
FITEM,2,201
FITEM,2,-290
SFBEAM,P51X,1,PRES,0.50,0.50, , , , ,
FLST,2,10,2,ORDE,2
FITEM,2,291
FITEM,2,-300
SFBEAM,P51X,1,PRES,0.25,0.25, , , , ,
!*
SAVE
/SOL
SOLVE
!*
/POST1
!*** ELEMENT PROPERTIS ***
!*
AVPRIN,0, ,
ETABLE,UX,U,X
AVPRIN,0, ,
ETABLE,UY,U,Y
AVPRIN,0, ,
ETABLE,PU,SMISC, 1
AVPRIN,0, ,
ETABLE,MI,SMISC, 6
AVPRIN,0, ,
ETABLE,MJ,SMISC, 12
PRETAB,UX,UY,PU,MI,MJ
!*
```

# APPENDIX-B

1- Full Catalog Section (FCS):

| No. | AISC Sections < 200 lb | Weigth (W) lb/ft | Area (A) $in^2$ | Depth (D) in | Width (BF) in | $t_w$ in | $t_f$ in | $b_f / 2t_f$ | $h / t_w$ | Ix $in^4$ | Zx $in^3$ | Sx $in^3$ | rx $in^3$ | Iy $in^4$ | Zy $in^3$ | Sy $in^3$ | ry $in^3$ | J | CW |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | W40X199 | 199 | 58.5 | 38.7 | 15.8 | 0.65 | 1.07 | 7.39 | 52.6 | 14900 | 869 | 770 | 16 | 695 | 137 | 88.2 | 3.45 | 18.3 | 246000 |
| 2 | W40X183 | 183 | 53.3 | 39 | 11.8 | 0.65 | 1.2 | 4.92 | 52.6 | 13200 | 774 | 675 | 15.7 | 331 | 88.3 | 56 | 2.49 | 19.3 | 118000 |
| 3 | W40X167 | 167 | 49.2 | 38.6 | 11.8 | 0.65 | 1.03 | 5.76 | 52.6 | 11600 | 693 | 600 | 15.3 | 283 | 76 | 47.9 | 2.4 | 14 | 99700 |
| 4 | W40X149 | 149 | 43.8 | 38.2 | 11.8 | 0.63 | 0.83 | 7.11 | 54.3 | 9800 | 598 | 513 | 15 | 229 | 62.2 | 38.8 | 2.29 | 9.36 | 80000 |
| 5 | W36X194 | 194 | 57 | 36.5 | 12.1 | 0.765 | 1.26 | 4.81 | 42.4 | 12100 | 767 | 664 | 14.6 | 375 | 97.7 | 61.9 | 2.56 | 22.2 | 116000 |
| 6 | W36X182 | 182 | 53.6 | 36.3 | 12.1 | 0.725 | 1.18 | 5.12 | 44.8 | 11300 | 718 | 623 | 14.5 | 347 | 90.7 | 57.6 | 2.55 | 18.5 | 107000 |
| 7 | W36X170 | 170 | 50.1 | 36.2 | 12 | 0.68 | 1.1 | 5.47 | 47.7 | 10500 | 668 | 581 | 14.5 | 320 | 83.8 | 53.2 | 2.53 | 15.1 | 98500 |
| 8 | W36X160 | 160 | 47 | 36 | 12 | 0.65 | 1.02 | 5.88 | 49.9 | 9760 | 624 | 542 | 14.4 | 295 | 77.3 | 49.1 | 2.5 | 12.4 | 90200 |
| 9 | W36X150 | 150 | 44.2 | 35.9 | 12 | 0.625 | 0.94 | 6.37 | 51.9 | 9040 | 581 | 504 | 14.3 | 270 | 70.9 | 45.1 | 2.47 | 10.1 | 82200 |
| 10 | W36X135 | 135 | 39.7 | 35.6 | 12 | 0.6 | 0.79 | 7.56 | 54.1 | 7800 | 509 | 439 | 14 | 225 | 59.7 | 37.7 | 2.38 | 7 | 68100 |
| 11 | W33X169 | 169 | 49.5 | 33.8 | 11.5 | 0.67 | 1.22 | 4.71 | 44.7 | 9290 | 629 | 549 | 13.7 | 310 | 84.4 | 53.9 | 2.5 | 17.7 | 82400 |
| 12 | W33X152 | 152 | 44.8 | 33.5 | 11.6 | 0.635 | 1.06 | 5.48 | 47.2 | 8160 | 559 | 487 | 13.5 | 273 | 73.9 | 47.2 | 2.47 | 12.4 | 71700 |
| 13 | W33X141 | 141 | 41.6 | 33.3 | 11.5 | 0.605 | 0.96 | 6.01 | 49.6 | 7450 | 514 | 448 | 13.4 | 246 | 66.9 | 42.7 | 2.43 | 9.7 | 64400 |
| 14 | W33X130 | 130 | 38.3 | 33.1 | 11.5 | 0.58 | 0.855 | 6.73 | 51.7 | 6710 | 467 | 406 | 13.2 | 218 | 59.5 | 37.9 | 2.39 | 7.37 | 56600 |
| 15 | W33X118 | 118 | 34.7 | 32.9 | 11.5 | 0.55 | 0.74 | 7.76 | 54.5 | 5900 | 415 | 359 | 13 | 187 | 51.3 | 32.6 | 2.32 | 5.3 | 48300 |
| 16 | W30X191 | 191 | 56.3 | 30.7 | 15 | 0.71 | 1.19 | 6.35 | 37.7 | 9200 | 675 | 600 | 12.8 | 673 | 138 | 89.5 | 3.46 | 21 | 146000 |
| 17 | W30X173 | 173 | 51 | 30.4 | 15 | 0.655 | 1.07 | 7.04 | 40.8 | 8230 | 607 | 541 | 12.7 | 598 | 123 | 79.8 | 3.42 | 15.6 | 129000 |
| 18 | W30X148 | 148 | 43.5 | 30.7 | 10.5 | 0.65 | 1.18 | 4.44 | 41.6 | 6680 | 500 | 436 | 12.4 | 227 | 68 | 43.3 | 2.28 | 14.5 | 49400 |
| 19 | W30X132 | 132 | 38.9 | 30.3 | 10.5 | 0.615 | 1 | 5.27 | 43.9 | 5770 | 437 | 380 | 12.2 | 196 | 58.4 | 37.2 | 2.25 | 9.72 | 42100 |
| 20 | W30X124 | 124 | 36.5 | 30.2 | 10.5 | 0.585 | 0.93 | 5.65 | 46.2 | 5360 | 408 | 355 | 12.1 | 181 | 54 | 34.4 | 2.23 | 7.99 | 38600 |
| 21 | W30X116 | 116 | 34.2 | 30 | 10.5 | 0.565 | 0.85 | 6.17 | 47.8 | 4930 | 378 | 329 | 12 | 164 | 49.2 | 31.3 | 2.19 | 6.43 | 34900 |
| 22 | W30X108 | 108 | 31.7 | 29.8 | 10.5 | 0.545 | 0.76 | 6.89 | 49.6 | 4470 | 346 | 299 | 11.9 | 146 | 43.9 | 27.9 | 2.15 | 4.99 | 30900 |
| 23 | W30X99 | 99 | 29.1 | 29.7 | 10.5 | 0.52 | 0.67 | 7.8 | 51.9 | 3990 | 312 | 269 | 11.7 | 128 | 38.6 | 24.5 | 2.1 | 3.77 | 26800 |
| 24 | W30X90 | 90 | 26.4 | 29.5 | 10.4 | 0.47 | 0.61 | 8.52 | 57.5 | 3610 | 283 | 245 | 11.7 | 115 | 34.7 | 22.1 | 2.09 | 2.84 | 24000 |
| 25 | W27X194 | 194 | 57.2 | 28.1 | 14 | 0.75 | 1.34 | 5.24 | 31.8 | 7860 | 631 | 559 | 11.7 | 619 | 136 | 88.1 | 3.29 | 27.1 | 111000 |
| 26 | W27X178 | 178 | 52.5 | 27.8 | 14.1 | 0.725 | 1.19 | 5.92 | 32.9 | 7020 | 570 | 505 | 11.6 | 555 | 122 | 78.8 | 3.25 | 20.1 | 98400 |

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 27 | W27X161 | 161 | 47.6 | 27.6 | 14 | 0.66 | 1.08 | 6.49 | 36.1 | 6310 | 515 | 458 | 11.5 | 497 | 109 | 70.9 | 3.23 | 15.1 | 87300 |
| 28 | W27X146 | 146 | 43.1 | 27.4 | 14 | 0.605 | 0.975 | 7.16 | 39.4 | 5660 | 464 | 414 | 11.5 | 443 | 97.7 | 63.5 | 3.2 | 11.3 | 77200 |
| 29 | W27X129 | 129 | 37.8 | 27.6 | 10 | 0.61 | 1.1 | 4.55 | 39.7 | 4760 | 395 | 345 | 11.2 | 184 | 57.6 | 36.8 | 2.21 | 11.1 | 32500 |
| 30 | W27X114 | 114 | 33.5 | 27.3 | 10.1 | 0.57 | 0.93 | 5.41 | 42.5 | 4080 | 343 | 299 | 11 | 159 | 49.3 | 31.5 | 2.18 | 7.33 | 27600 |
| 31 | W27X102 | 102 | 30 | 27.1 | 10 | 0.515 | 0.83 | 6.03 | 47.1 | 3620 | 305 | 267 | 11 | 139 | 43.4 | 27.8 | 2.15 | 5.28 | 24000 |
| 32 | W27X94 | 94 | 27.7 | 26.9 | 10 | 0.49 | 0.745 | 6.7 | 49.5 | 3270 | 278 | 243 | 10.9 | 124 | 38.8 | 24.8 | 2.12 | 4.03 | 21300 |
| 33 | W27X84 | 84 | 24.8 | 26.7 | 10 | 0.46 | 0.64 | 7.78 | 52.7 | 2850 | 244 | 213 | 10.7 | 106 | 33.2 | 21.2 | 2.07 | 2.81 | 17900 |
| 34 | W24X192 | 192 | 56.3 | 25.5 | 13 | 0.81 | 1.46 | 4.43 | 26.6 | 6260 | 559 | 491 | 10.5 | 530 | 126 | 81.8 | 3.07 | 30.8 | 76300 |
| 35 | W24X176 | 176 | 51.7 | 25.2 | 12.9 | 0.75 | 1.34 | 4.81 | 28.7 | 5680 | 511 | 450 | 10.5 | 479 | 115 | 74.3 | 3.04 | 23.9 | 68400 |
| 36 | W24X162 | 162 | 47.7 | 25 | 13 | 0.705 | 1.22 | 5.31 | 30.6 | 5170 | 468 | 414 | 10.4 | 443 | 105 | 68.4 | 3.05 | 18.5 | 62600 |
| 37 | W24X146 | 146 | 43 | 24.7 | 12.9 | 0.65 | 1.09 | 5.92 | 33.2 | 4580 | 418 | 371 | 10.3 | 391 | 93.2 | 60.5 | 3.01 | 13.4 | 54600 |
| 38 | W24X131 | 131 | 38.5 | 24.5 | 12.9 | 0.605 | 0.96 | 6.7 | 35.6 | 4020 | 370 | 329 | 10.2 | 340 | 81.5 | 53 | 2.97 | 9.5 | 47100 |
| 39 | W24X117 | 117 | 34.4 | 24.3 | 12.8 | 0.55 | 0.85 | 7.53 | 39.2 | 3540 | 327 | 291 | 10.1 | 297 | 71.4 | 46.5 | 2.94 | 6.72 | 40800 |
| 40 | W24X104 | 104 | 30.6 | 24.1 | 12.8 | 0.5 | 0.75 | 8.5 | 43.1 | 3100 | 289 | 258 | 10.1 | 259 | 62.4 | 40.7 | 2.91 | 4.72 | 35200 |
| 41 | W24X103 | 103 | 30.3 | 24.5 | 9 | 0.55 | 0.98 | 4.59 | 39.2 | 3000 | 280 | 245 | 10 | 119 | 41.5 | 26.5 | 1.99 | 7.07 | 16600 |
| 42 | W24X94 | 94 | 27.7 | 24.3 | 9.07 | 0.515 | 0.875 | 5.18 | 41.9 | 2700 | 254 | 222 | 9.87 | 109 | 37.5 | 24 | 1.98 | 5.26 | 15000 |
| 43 | W24X84 | 84 | 24.7 | 24.1 | 9.02 | 0.47 | 0.77 | 5.86 | 45.9 | 2370 | 224 | 196 | 9.79 | 94.4 | 32.6 | 20.9 | 1.95 | 3.7 | 12800 |
| 44 | W24X76 | 76 | 22.4 | 23.9 | 8.99 | 0.44 | 0.68 | 6.61 | 49 | 2100 | 200 | 176 | 9.69 | 82.5 | 28.6 | 18.4 | 1.92 | 2.68 | 11100 |
| 45 | W24X68 | 68 | 20.1 | 23.7 | 8.97 | 0.415 | 0.585 | 7.66 | 52 | 1830 | 177 | 154 | 9.55 | 70.4 | 24.5 | 15.7 | 1.87 | 1.87 | 9430 |
| 46 | W24X62 | 62 | 18.2 | 23.7 | 7.04 | 0.43 | 0.59 | 5.97 | 50.1 | 1550 | 153 | 131 | 9.23 | 34.5 | 15.7 | 9.8 | 1.38 | 1.71 | 4620 |
| 47 | W24X55 | 55 | 16.2 | 23.6 | 7.01 | 0.395 | 0.505 | 6.94 | 54.6 | 1350 | 134 | 114 | 9.11 | 29.1 | 13.3 | 8.3 | 1.34 | 1.18 | 3870 |
| 48 | W21X182 | 182 | 53.6 | 22.7 | 12.5 | 0.83 | 1.48 | 4.22 | 22.6 | 4730 | 476 | 417 | 9.4 | 483 | 119 | 77.2 | 3 | 30.7 | 54400 |
| 49 | W21X166 | 166 | 48.8 | 22.5 | 12.4 | 0.75 | 1.36 | 4.57 | 25 | 4280 | 432 | 380 | 9.36 | 435 | 108 | 70 | 2.99 | 23.6 | 48500 |
| 50 | W21X147 | 147 | 43.2 | 22.1 | 12.5 | 0.72 | 1.15 | 5.44 | 26.1 | 3630 | 373 | 329 | 9.17 | 376 | 92.6 | 60.1 | 2.95 | 15.4 | 41100 |
| 51 | W21X132 | 132 | 38.8 | 21.8 | 12.4 | 0.65 | 1.04 | 6.01 | 28.9 | 3220 | 333 | 295 | 9.12 | 333 | 82.3 | 53.5 | 2.93 | 11.3 | 36000 |
| 52 | W21X122 | 122 | 35.9 | 21.7 | 12.4 | 0.6 | 0.96 | 6.45 | 31.3 | 2960 | 307 | 273 | 9.09 | 305 | 75.6 | 49.2 | 2.92 | 8.98 | 32700 |
| 53 | W21X111 | 111 | 32.7 | 21.5 | 12.3 | 0.55 | 0.875 | 7.05 | 34.1 | 2670 | 279 | 249 | 9.05 | 274 | 68.2 | 44.5 | 2.9 | 6.83 | 29200 |
| 54 | W21X101 | 101 | 29.8 | 21.4 | 12.3 | 0.5 | 0.8 | 7.68 | 37.5 | 2420 | 253 | 227 | 9.02 | 248 | 61.7 | 40.3 | 2.89 | 5.21 | 26200 |
| 55 | W21X93 | 93 | 27.3 | 21.6 | 8.42 | 0.58 | 0.93 | 4.53 | 32.3 | 2070 | 221 | 192 | 8.7 | 92.9 | 34.7 | 22.1 | 1.84 | 6.03 | 9940 |
| 56 | W21X83 | 83 | 24.3 | 21.4 | 8.36 | 0.515 | 0.835 | 5 | 36.4 | 1830 | 196 | 171 | 8.67 | 81.4 | 30.5 | 19.5 | 1.83 | 4.34 | 8630 |
| 57 | W21X73 | 73 | 21.5 | 21.2 | 8.3 | 0.455 | 0.74 | 5.6 | 41.2 | 1600 | 172 | 151 | 8.64 | 70.6 | 26.6 | 17 | 1.81 | 3.02 | 7410 |
| 58 | W21X68 | 68 | 20 | 21.1 | 8.27 | 0.43 | 0.685 | 6.04 | 43.6 | 1480 | 160 | 140 | 8.6 | 64.7 | 24.4 | 15.7 | 1.8 | 2.45 | 6760 |
| 59 | W21X62 | 62 | 18.3 | 21 | 8.24 | 0.4 | 0.615 | 6.7 | 46.9 | 1330 | 144 | 127 | 8.54 | 57.5 | 21.7 | 14 | 1.77 | 1.83 | 5960 |
| 60 | W21X55 | 55 | 16.2 | 20.8 | 8.22 | 0.375 | 0.522 | 7.87 | 50 | 1140 | 126 | 110 | 8.4 | 48.4 | 18.4 | 11.8 | 1.73 | 1.24 | 4980 |
| 61 | W21X48 | 48 | 14.1 | 20.6 | 8.14 | 0.35 | 0.43 | 9.47 | 53.6 | 959 | 107 | 93 | 8.24 | 38.7 | 14.9 | 9.52 | 1.66 | 0.803 | 3950 |

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 62 | W21X57 | 57 | 16.7 | 21.1 | 6.56 | 0.405 | 0.65 | 5.04 | 46.3 | 1170 | 129 | 111 | 8.36 | 30.6 | 14.8 | 9.35 | 1.35 | 1.77 | 3190 |
| 63 | W21X50 | 50 | 14.7 | 20.8 | 6.53 | 0.38 | 0.535 | 6.1 | 49.4 | 984 | 110 | 94.5 | 8.18 | 24.9 | 12.2 | 7.64 | 1.3 | 1.14 | 2570 |
| 64 | W21X44 | 44 | 13 | 20.7 | 6.5 | 0.35 | 0.45 | 7.22 | 53.6 | 843 | 95.4 | 81.6 | 8.06 | 20.7 | 10.2 | 6.37 | 1.26 | 0.77 | 2110 |
| 65 | W18x192 | 192 | 56.4 | 20.4 | 11.5 | 0.96 | 1.75 | 3.27 | 16.7 | 3870 | 442 | 380 | 8.28 | 440 | 119 | 76.8 | 2.79 | 44.7 | 38000 |
| 66 | W18X175 | 175 | 51.3 | 20 | 11.4 | 0.89 | 1.59 | 3.58 | 18 | 3450 | 398 | 344 | 8.2 | 391 | 106 | 68.8 | 2.76 | 33.8 | 33300 |
| 67 | W18X158 | 158 | 46.3 | 19.7 | 11.3 | 0.81 | 1.44 | 3.92 | 19.8 | 3060 | 356 | 310 | 8.12 | 347 | 94.8 | 61.4 | 2.74 | 25.2 | 29000 |
| 68 | W18X143 | 143 | 42.1 | 19.5 | 11.2 | 0.73 | 1.32 | 4.25 | 22 | 2750 | 322 | 282 | 8.09 | 311 | 85.4 | 55.5 | 2.72 | 19.2 | 25700 |
| 69 | W18X130 | 130 | 38.2 | 19.3 | 11.2 | 0.67 | 1.2 | 4.65 | 23.9 | 2460 | 290 | 256 | 8.03 | 278 | 76.7 | 49.9 | 2.7 | 14.5 | 22700 |
| 70 | W18X119 | 119 | 35.1 | 19 | 11.3 | 0.655 | 1.06 | 5.31 | 24.5 | 2190 | 262 | 231 | 7.9 | 253 | 69.1 | 44.9 | 2.69 | 10.6 | 20300 |
| 71 | W18X106 | 106 | 31.1 | 18.7 | 11.2 | 0.59 | 0.94 | 5.96 | 27.2 | 1910 | 230 | 204 | 7.84 | 220 | 60.5 | 39.4 | 2.66 | 7.48 | 17400 |
| 72 | W18X97 | 97 | 28.5 | 18.6 | 11.1 | 0.535 | 0.87 | 6.41 | 30 | 1750 | 211 | 188 | 7.82 | 201 | 55.3 | 36.1 | 2.65 | 5.86 | 15800 |
| 73 | W18X86 | 86 | 25.3 | 18.4 | 11.1 | 0.48 | 0.77 | 7.2 | 33.4 | 1530 | 186 | 166 | 7.77 | 175 | 48.4 | 31.6 | 2.63 | 4.1 | 13600 |
| 74 | W18X76 | 76 | 22.3 | 18.2 | 11 | 0.425 | 0.68 | 8.11 | 37.8 | 1330 | 163 | 146 | 7.73 | 152 | 42.2 | 27.6 | 2.61 | 2.83 | 11700 |
| 75 | W18X71 | 71 | 20.8 | 18.5 | 7.64 | 0.495 | 0.81 | 4.71 | 32.4 | 1170 | 146 | 127 | 7.5 | 60.3 | 24.7 | 15.8 | 1.7 | 3.49 | 4700 |
| 76 | W18X65 | 65 | 19.1 | 18.4 | 7.59 | 0.45 | 0.75 | 5.06 | 35.7 | 1070 | 133 | 117 | 7.49 | 54.8 | 22.5 | 14.4 | 1.69 | 2.73 | 4240 |
| 77 | W18X60 | 60 | 17.6 | 18.2 | 7.56 | 0.415 | 0.695 | 5.44 | 38.7 | 984 | 123 | 108 | 7.47 | 50.1 | 20.6 | 13.3 | 1.68 | 2.17 | 3850 |
| 78 | W18X55 | 55 | 16.2 | 18.1 | 7.53 | 0.39 | 0.63 | 5.98 | 41.1 | 890 | 112 | 98.3 | 7.41 | 44.9 | 18.5 | 11.9 | 1.67 | 1.66 | 3430 |
| 79 | W18X50 | 50 | 14.7 | 18 | 7.5 | 0.355 | 0.57 | 6.57 | 45.2 | 800 | 101 | 88.9 | 7.38 | 40.1 | 16.6 | 10.7 | 1.65 | 1.24 | 3040 |
| 80 | W18X46 | 46 | 13.5 | 18.1 | 6.06 | 0.36 | 0.605 | 5.01 | 44.6 | 712 | 90.7 | 78.8 | 7.25 | 22.5 | 11.7 | 7.43 | 1.29 | 1.22 | 1720 |
| 81 | W18X40 | 40 | 11.8 | 17.9 | 6.02 | 0.315 | 0.525 | 5.73 | 50.9 | 612 | 78.4 | 68.4 | 7.21 | 19.1 | 10 | 6.35 | 1.27 | 0.81 | 1440 |
| 82 | W18X35 | 35 | 10.3 | 17.7 | 6 | 0.3 | 0.425 | 7.06 | 53.5 | 510 | 66.5 | 57.6 | 7.04 | 15.3 | 8.06 | 5.12 | 1.22 | 0.506 | 1140 |
| 83 | W16X100 | 100 | 29.5 | 17 | 10.4 | 0.585 | 0.985 | 5.29 | 24.3 | 1490 | 198 | 175 | 7.1 | 186 | 54.9 | 35.7 | 2.51 | 7.73 | 11900 |
| 84 | W16X89 | 89 | 26.2 | 16.8 | 10.4 | 0.525 | 0.875 | 5.92 | 27 | 1300 | 175 | 155 | 7.05 | 163 | 48.1 | 31.4 | 2.49 | 5.45 | 10200 |
| 85 | W16X77 | 77 | 22.6 | 16.5 | 10.3 | 0.455 | 0.76 | 6.77 | 31.2 | 1110 | 150 | 134 | 7 | 138 | 41.1 | 26.9 | 2.47 | 3.57 | 8590 |
| 86 | W16X67 | 67 | 19.7 | 16.3 | 10.2 | 0.395 | 0.665 | 7.7 | 35.9 | 954 | 130 | 117 | 6.96 | 119 | 35.5 | 23.2 | 2.46 | 2.39 | 7300 |
| 87 | W16X57 | 57 | 16.8 | 16.4 | 7.12 | 0.43 | 0.715 | 4.98 | 33 | 758 | 105 | 92.2 | 6.72 | 43.1 | 18.9 | 12.1 | 1.6 | 2.22 | 2660 |
| 88 | W16X50 | 50 | 14.7 | 16.3 | 7.07 | 0.38 | 0.63 | 5.61 | 37.4 | 659 | 92 | 81 | 6.68 | 37.2 | 16.3 | 10.5 | 1.59 | 1.52 | 2270 |
| 89 | W16X45 | 45 | 13.3 | 16.1 | 7.04 | 0.345 | 0.565 | 6.23 | 41.1 | 586 | 82.3 | 72.7 | 6.65 | 32.8 | 14.5 | 9.34 | 1.57 | 1.11 | 1990 |
| 90 | W16X40 | 40 | 11.8 | 16 | 7 | 0.305 | 0.505 | 6.93 | 46.5 | 518 | 73 | 64.7 | 6.63 | 28.9 | 12.7 | 8.25 | 1.57 | 0.794 | 1730 |
| 91 | W16X36 | 36 | 10.6 | 15.9 | 6.99 | 0.295 | 0.43 | 8.12 | 48.1 | 448 | 64 | 56.5 | 6.51 | 24.5 | 10.8 | 7 | 1.52 | 0.545 | 1460 |
| 92 | W16X31 | 31 | 9.13 | 15.9 | 5.53 | 0.275 | 0.44 | 6.28 | 51.6 | 375 | 54 | 47.2 | 6.41 | 12.4 | 7.03 | 4.49 | 1.17 | 0.461 | 739 |
| 93 | W16X26 | 26 | 7.68 | 15.7 | 5.5 | 0.25 | 0.345 | 7.97 | 56.8 | 301 | 44.2 | 38.4 | 6.26 | 9.59 | 5.48 | 3.49 | 1.12 | 0.262 | 565 |
| 94 | W14X193 | 193 | 56.8 | 15.5 | 15.7 | 0.89 | 1.44 | 5.45 | 12.8 | 2400 | 355 | 310 | 6.5 | 931 | 180 | 119 | 4.05 | 34.8 | 45900 |
| 95 | W14X176 | 176 | 51.8 | 15.2 | 15.7 | 0.83 | 1.31 | 5.97 | 13.7 | 2140 | 320 | 281 | 6.43 | 838 | 163 | 107 | 4.02 | 26.5 | 40500 |
| 96 | W14X159 | 159 | 46.7 | 15 | 15.6 | 0.745 | 1.19 | 6.54 | 15.3 | 1900 | 287 | 254 | 6.38 | 748 | 146 | 96.2 | 4 | 19.7 | 35600 |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 97 | W14X145 | 145 | 42.7 | 14.8 | 15.5 | 0.68 | 1.09 | 7.11 | 16.8 | 1710 | 260 | 232 | 6.33 | 677 | 133 | 87.3 | 3.98 | 15.2 | 31700 |
| 98 | W14X132 | 132 | 38.8 | 14.7 | 14.7 | 0.645 | 1.03 | 7.15 | 17.7 | 1530 | 234 | 209 | 6.28 | 548 | 113 | 74.5 | 3.76 | 12.3 | 25500 |
| 99 | W14X120 | 120 | 35.3 | 14.5 | 14.7 | 0.59 | 0.94 | 7.8 | 19.3 | 1380 | 212 | 190 | 6.24 | 495 | 102 | 67.5 | 3.74 | 9.37 | 22700 |
| 100 | W14X109 | 109 | 32 | 14.3 | 14.6 | 0.525 | 0.86 | 8.49 | 21.7 | 1240 | 192 | 173 | 6.22 | 447 | 92.7 | 61.2 | 3.73 | 7.12 | 20200 |
| 101 | W14X99 | 99 | 29.1 | 14.2 | 14.6 | 0.485 | 0.78 | 9.34 | 23.5 | 1110 | 173 | 157 | 6.17 | 402 | 83.6 | 55.2 | 3.71 | 5.37 | 18000 |
| 102 | W14X90 | 90 | 26.5 | 14 | 14.5 | 0.44 | 0.71 | 10.2 | 25.9 | 999 | 157 | 143 | 6.14 | 362 | 75.6 | 49.9 | 3.7 | 4.06 | 16000 |
| 103 | W14X82 | 82 | 24 | 14.3 | 10.1 | 0.51 | 0.855 | 5.92 | 22.4 | 881 | 139 | 123 | 6.05 | 148 | 44.8 | 29.3 | 2.48 | 5.07 | 6710 |
| 104 | W14X74 | 74 | 21.8 | 14.2 | 10.1 | 0.45 | 0.785 | 6.41 | 25.4 | 795 | 126 | 112 | 6.04 | 134 | 40.5 | 26.6 | 2.48 | 3.87 | 5990 |
| 105 | W14X68 | 68 | 20 | 14 | 10 | 0.415 | 0.72 | 6.97 | 27.5 | 722 | 115 | 103 | 6.01 | 121 | 36.9 | 24.2 | 2.46 | 3.01 | 5380 |
| 106 | W14X61 | 61 | 17.9 | 13.9 | 10 | 0.375 | 0.645 | 7.75 | 30.4 | 640 | 102 | 92.1 | 5.98 | 107 | 32.8 | 21.5 | 2.45 | 2.19 | 4710 |
| 107 | W14X53 | 53 | 15.6 | 13.9 | 8.06 | 0.37 | 0.66 | 6.11 | 30.9 | 541 | 87.1 | 77.8 | 5.89 | 57.7 | 22 | 14.3 | 1.92 | 1.94 | 2540 |
| 108 | W14X48 | 48 | 14.1 | 13.8 | 8.03 | 0.34 | 0.595 | 6.75 | 33.6 | 484 | 78.4 | 70.2 | 5.85 | 51.4 | 19.6 | 12.8 | 1.91 | 1.45 | 2240 |
| 109 | W14X43 | 43 | 12.6 | 13.7 | 8 | 0.305 | 0.53 | 7.54 | 37.4 | 428 | 69.6 | 62.6 | 5.82 | 45.2 | 17.3 | 11.3 | 1.89 | 1.05 | 1950 |
| 110 | W14X38 | 38 | 11.2 | 14.1 | 6.77 | 0.31 | 0.515 | 6.57 | 39.6 | 385 | 61.5 | 54.6 | 5.87 | 26.7 | 12.1 | 7.88 | 1.55 | 0.798 | 1230 |
| 111 | W14X34 | 34 | 10 | 14 | 6.75 | 0.285 | 0.455 | 7.41 | 43.1 | 340 | 54.6 | 48.6 | 5.83 | 23.3 | 10.6 | 6.91 | 1.53 | 0.569 | 1070 |
| 112 | W14X30 | 30 | 8.85 | 13.8 | 6.73 | 0.27 | 0.385 | 8.74 | 45.4 | 291 | 47.3 | 42 | 5.73 | 19.6 | 8.99 | 5.82 | 1.49 | 0.38 | 887 |
| 113 | W14X26 | 26 | 7.69 | 13.9 | 5.03 | 0.255 | 0.42 | 5.98 | 48.1 | 245 | 40.2 | 35.3 | 5.65 | 8.91 | 5.54 | 3.55 | 1.08 | 0.358 | 405 |
| 114 | W14X22 | 22 | 6.49 | 13.7 | 5 | 0.23 | 0.335 | 7.46 | 53.3 | 199 | 33.2 | 29 | 5.54 | 7 | 4.39 | 2.8 | 1.04 | 0.208 | 314 |
| 115 | W12X190 | 190 | 55.8 | 14.4 | 12.7 | 1.06 | 1.74 | 3.65 | 9.16 | 1890 | 311 | 263 | 5.82 | 589 | 143 | 93 | 3.25 | 48.8 | 23600 |
| 116 | W12X170 | 170 | 50 | 14 | 12.6 | 0.96 | 1.56 | 4.03 | 10.1 | 1650 | 275 | 235 | 5.74 | 517 | 126 | 82.3 | 3.22 | 35.6 | 20100 |
| 117 | W12X152 | 152 | 44.7 | 13.7 | 12.5 | 0.87 | 1.4 | 4.46 | 11.2 | 1430 | 243 | 209 | 5.66 | 454 | 111 | 72.8 | 3.19 | 25.8 | 17200 |
| 118 | W12X136 | 136 | 39.9 | 13.4 | 12.4 | 0.79 | 1.25 | 4.96 | 12.3 | 1240 | 214 | 186 | 5.58 | 398 | 98 | 64.2 | 3.16 | 18.5 | 14700 |
| 119 | W12X120 | 120 | 35.3 | 13.1 | 12.3 | 0.71 | 1.11 | 5.57 | 13.7 | 1070 | 186 | 163 | 5.51 | 345 | 85.4 | 56 | 3.13 | 12.9 | 12400 |
| 120 | W12X106 | 106 | 31.2 | 12.9 | 12.2 | 0.61 | 0.99 | 6.17 | 15.9 | 933 | 164 | 145 | 5.47 | 301 | 75.1 | 49.3 | 3.11 | 9.13 | 10700 |
| 121 | W12X96 | 96 | 28.2 | 12.7 | 12.2 | 0.55 | 0.9 | 6.76 | 17.7 | 833 | 147 | 131 | 5.44 | 270 | 67.5 | 44.4 | 3.09 | 6.85 | 9410 |
| 122 | W12X87 | 87 | 25.6 | 12.5 | 12.1 | 0.515 | 0.81 | 7.48 | 18.9 | 740 | 132 | 118 | 5.38 | 241 | 60.4 | 39.7 | 3.07 | 5.1 | 8270 |
| 123 | W12X79 | 79 | 23.2 | 12.4 | 12.1 | 0.47 | 0.735 | 8.22 | 20.7 | 662 | 119 | 107 | 5.34 | 216 | 54.3 | 35.8 | 3.05 | 3.84 | 7330 |
| 124 | W12X72 | 72 | 21.1 | 12.3 | 12 | 0.43 | 0.67 | 8.99 | 22.6 | 597 | 108 | 97.4 | 5.31 | 195 | 49.2 | 32.4 | 3.04 | 2.93 | 6540 |
| 125 | W12X65 | 65 | 19.1 | 12.1 | 12 | 0.39 | 0.605 | 9.92 | 24.9 | 533 | 96.8 | 87.9 | 5.28 | 174 | 44.1 | 29.1 | 3.02 | 2.18 | 5780 |
| 126 | W12X58 | 58 | 17 | 12.2 | 10 | 0.36 | 0.64 | 7.82 | 27 | 475 | 86.4 | 78 | 5.28 | 107 | 32.5 | 21.4 | 2.51 | 2.1 | 3570 |
| 127 | W12X53 | 53 | 15.6 | 12.1 | 10 | 0.345 | 0.575 | 8.69 | 28.1 | 425 | 77.9 | 70.6 | 5.23 | 95.8 | 29.1 | 19.2 | 2.48 | 1.58 | 3160 |
| 128 | W12X50 | 50 | 14.6 | 12.2 | 8.08 | 0.37 | 0.64 | 6.31 | 26.8 | 391 | 71.9 | 64.2 | 5.18 | 56.3 | 21.3 | 13.9 | 1.96 | 1.71 | 1880 |
| 129 | W12X45 | 45 | 13.1 | 12.1 | 8.05 | 0.335 | 0.575 | 7 | 29.6 | 348 | 64.2 | 57.7 | 5.15 | 50 | 19 | 12.4 | 1.95 | 1.26 | 1650 |
| 130 | W12X40 | 40 | 11.7 | 11.9 | 8.01 | 0.295 | 0.515 | 7.77 | 33.6 | 307 | 57 | 51.5 | 5.13 | 44.1 | 16.8 | 11 | 1.94 | 0.906 | 1440 |
| 131 | W12X35 | 35 | 10.3 | 12.5 | 6.56 | 0.3 | 0.52 | 6.31 | 36.2 | 285 | 51.2 | 45.6 | 5.25 | 24.5 | 11.5 | 7.47 | 1.54 | 0.741 | 879 |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 132 | W12X30 | 30 | 8.79 | 12.3 | 6.52 | 0.26 | 0.44 | 7.41 | 41.8 | 238 | 43.1 | 38.6 | 5.21 | 20.3 | 9.56 | 6.24 | 1.52 | 0.457 | 720 |
| 133 | W12X26 | 26 | 7.65 | 12.2 | 6.49 | 0.23 | 0.38 | 8.54 | 47.2 | 204 | 37.2 | 33.4 | 5.17 | 17.3 | 8.17 | 5.34 | 1.51 | 0.3 | 607 |
| 134 | W12X22 | 22 | 6.48 | 12.3 | 4.03 | 0.26 | 0.425 | 4.74 | 41.8 | 156 | 29.3 | 25.4 | 4.91 | 4.66 | 3.66 | 2.31 | 0.848 | 0.293 | 164 |
| 135 | W12X19 | 19 | 5.57 | 12.2 | 4.01 | 0.235 | 0.35 | 5.72 | 46.2 | 130 | 24.7 | 21.3 | 4.82 | 3.76 | 2.98 | 1.88 | 0.822 | 0.18 | 131 |
| 136 | W12X16 | 16 | 4.71 | 12 | 3.99 | 0.22 | 0.265 | 7.53 | 49.4 | 103 | 20.1 | 17.1 | 4.67 | 2.82 | 2.26 | 1.41 | 0.773 | 0.103 | 96.9 |
| 137 | W12X14 | 14 | 4.16 | 11.9 | 3.97 | 0.2 | 0.225 | 8.82 | 54.3 | 88.6 | 17.4 | 14.9 | 4.62 | 2.36 | 1.9 | 1.19 | 0.753 | 0.0704 | 80.4 |
| 138 | W10X112 | 112 | 32.9 | 11.4 | 10.4 | 0.755 | 1.25 | 4.17 | 10.4 | 716 | 147 | 126 | 4.66 | 236 | 69.2 | 45.3 | 2.68 | 15.1 | 6020 |
| 139 | W10X100 | 100 | 29.4 | 11.1 | 10.3 | 0.68 | 1.12 | 4.62 | 11.6 | 623 | 130 | 112 | 4.6 | 207 | 61 | 40 | 2.65 | 10.9 | 5150 |
| 140 | W10X88 | 88 | 25.9 | 10.8 | 10.3 | 0.605 | 0.99 | 5.18 | 13 | 534 | 113 | 98.5 | 4.54 | 179 | 53.1 | 34.8 | 2.63 | 7.53 | 4330 |
| 141 | W10X77 | 77 | 22.6 | 10.6 | 10.2 | 0.53 | 0.87 | 5.86 | 14.8 | 455 | 97.6 | 85.9 | 4.49 | 154 | 45.9 | 30.1 | 2.6 | 5.11 | 3630 |
| 142 | W10X68 | 68 | 20 | 10.4 | 10.1 | 0.47 | 0.77 | 6.58 | 16.7 | 394 | 85.3 | 75.7 | 4.44 | 134 | 40.1 | 26.4 | 2.59 | 3.56 | 3100 |
| 143 | W10X60 | 60 | 17.6 | 10.2 | 10.1 | 0.42 | 0.68 | 7.41 | 18.7 | 341 | 74.6 | 66.7 | 4.39 | 116 | 35 | 23 | 2.57 | 2.48 | 2640 |
| 144 | W10X54 | 54 | 15.8 | 10.1 | 10 | 0.37 | 0.615 | 8.15 | 21.2 | 303 | 66.6 | 60 | 4.37 | 103 | 31.3 | 20.6 | 2.56 | 1.82 | 2320 |
| 145 | W10X49 | 49 | 14.4 | 10 | 10 | 0.34 | 0.56 | 8.93 | 23.1 | 272 | 60.4 | 54.6 | 4.35 | 93.4 | 28.3 | 18.7 | 2.54 | 1.39 | 2070 |
| 146 | W10X45 | 45 | 13.3 | 10.1 | 8.02 | 0.35 | 0.62 | 6.47 | 22.5 | 248 | 54.9 | 49.1 | 4.32 | 53.4 | 20.3 | 13.3 | 2.01 | 1.51 | 1200 |
| 147 | W10X39 | 39 | 11.5 | 9.92 | 7.99 | 0.315 | 0.53 | 7.53 | 25 | 209 | 46.8 | 42.1 | 4.27 | 45 | 17.2 | 11.3 | 1.98 | 0.976 | 992 |
| 148 | W10X33 | 33 | 9.71 | 9.73 | 7.96 | 0.29 | 0.435 | 9.15 | 27.1 | 171 | 38.8 | 35 | 4.19 | 36.6 | 14 | 9.2 | 1.94 | 0.583 | 791 |
| 149 | W10X30 | 30 | 8.84 | 10.5 | 5.81 | 0.3 | 0.51 | 5.7 | 29.5 | 170 | 36.6 | 32.4 | 4.38 | 16.7 | 8.84 | 5.75 | 1.37 | 0.622 | 414 |
| 150 | W10X26 | 26 | 7.61 | 10.3 | 5.77 | 0.26 | 0.44 | 6.56 | 34 | 144 | 31.3 | 27.9 | 4.35 | 14.1 | 7.5 | 4.89 | 1.36 | 0.402 | 345 |
| 151 | W10X22 | 22 | 6.49 | 10.2 | 5.75 | 0.24 | 0.36 | 7.99 | 36.9 | 118 | 26 | 23.2 | 4.27 | 11.4 | 6.1 | 3.97 | 1.33 | 0.239 | 275 |
| 152 | W10X19 | 19 | 5.62 | 10.2 | 4.02 | 0.25 | 0.395 | 5.09 | 35.4 | 96.3 | 21.6 | 18.8 | 4.14 | 4.29 | 3.35 | 2.14 | 0.874 | 0.233 | 104 |
| 153 | W10X17 | 17 | 4.99 | 10.1 | 4.01 | 0.24 | 0.33 | 6.08 | 36.9 | 81.9 | 18.7 | 16.2 | 4.05 | 3.56 | 2.8 | 1.78 | 0.845 | 0.156 | 85.1 |
| 154 | W10X15 | 15 | 4.41 | 10 | 4 | 0.23 | 0.27 | 7.41 | 38.5 | 68.9 | 16 | 13.8 | 3.95 | 2.89 | 2.3 | 1.45 | 0.81 | 0.104 | 68.3 |
| 155 | W10X12 | 12 | 3.54 | 9.87 | 3.96 | 0.19 | 0.21 | 9.43 | 46.6 | 53.8 | 12.6 | 10.9 | 3.9 | 2.18 | 1.74 | 1.1 | 0.785 | 0.0547 | 50.9 |
| 156 | W8X67 | 67 | 19.7 | 9 | 8.28 | 0.57 | 0.935 | 4.43 | 11.1 | 272 | 70.1 | 60.4 | 3.72 | 88.6 | 32.7 | 21.4 | 2.12 | 5.05 | 1440 |
| 157 | W8X58 | 58 | 17.1 | 8.75 | 8.22 | 0.51 | 0.81 | 5.07 | 12.4 | 228 | 59.8 | 52 | 3.65 | 75.1 | 27.9 | 18.3 | 2.1 | 3.33 | 1180 |
| 158 | W8X48 | 48 | 14.1 | 8.5 | 8.11 | 0.4 | 0.685 | 5.92 | 15.9 | 184 | 49 | 43.2 | 3.61 | 60.9 | 22.9 | 15 | 2.08 | 1.96 | 931 |
| 159 | W8X40 | 40 | 11.7 | 8.25 | 8.07 | 0.36 | 0.56 | 7.21 | 17.6 | 146 | 39.8 | 35.5 | 3.53 | 49.1 | 18.5 | 12.2 | 2.04 | 1.12 | 726 |
| 160 | W8X35 | 35 | 10.3 | 8.12 | 8.02 | 0.31 | 0.495 | 8.1 | 20.5 | 127 | 34.7 | 31.2 | 3.51 | 42.6 | 16.1 | 10.6 | 2.03 | 0.769 | 619 |
| 161 | W8X31 | 31 | 9.12 | 8 | 8 | 0.285 | 0.435 | 9.19 | 22.3 | 110 | 30.4 | 27.5 | 3.47 | 37.1 | 14.1 | 9.27 | 2.02 | 0.536 | 530 |
| 162 | W8X28 | 28 | 8.24 | 8.06 | 6.54 | 0.285 | 0.465 | 7.03 | 22.3 | 98 | 27.2 | 24.3 | 3.45 | 21.7 | 10.1 | 6.63 | 1.62 | 0.537 | 312 |
| 163 | W8X24 | 24 | 7.08 | 7.93 | 6.5 | 0.245 | 0.4 | 8.12 | 25.9 | 82.7 | 23.1 | 20.9 | 3.42 | 18.3 | 8.57 | 5.63 | 1.61 | 0.346 | 259 |
| 164 | W8X21 | 21 | 6.16 | 8.28 | 5.27 | 0.25 | 0.4 | 6.59 | 27.5 | 75.3 | 20.4 | 18.2 | 3.49 | 9.77 | 5.69 | 3.71 | 1.26 | 0.282 | 152 |
| 165 | W8X18 | 18 | 5.26 | 8.14 | 5.25 | 0.23 | 0.33 | 7.95 | 29.9 | 61.9 | 17 | 15.2 | 3.43 | 7.97 | 4.66 | 3.04 | 1.23 | 0.172 | 122 |
| 166 | W8X15 | 15 | 4.44 | 8.11 | 4.01 | 0.245 | 0.315 | 6.37 | 28.1 | 48 | 13.6 | 11.8 | 3.29 | 3.41 | 2.67 | 1.7 | 0.876 | 0.137 | 51.8 |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 167 | W8X13 | 13 | 3.84 | 7.99 | 4 | 0.23 | 0.255 | 7.84 | 29.9 | 39.6 | 11.4 | 9.91 | 3.21 | 2.73 | 2.15 | 1.37 | 0.843 | 0.0871 | 40.8 |
| 168 | W8X10 | 10 | 2.96 | 7.89 | 3.94 | 0.17 | 0.205 | 9.61 | 40.5 | 30.8 | 8.87 | 7.81 | 3.22 | 2.09 | 1.66 | 1.06 | 0.841 | 0.0426 | 30.9 |

## 2- Selected Catalog Section (SCS):

### 2.1 Column catalog sections.

| No. | AISC Sections < 200 lb | Weight (W) lb/ft | Area (A) $in^2$ | Depth (D) in | Width (BF) in | $t_w$ in | $t_f$ in | $b_f / 2t_f$ | $h / t_w$ | Ix $in^4$ | Zx $in^3$ | Sx $in^3$ | rx $in^3$ | Iy $in^4$ | Zy $in^3$ | Sy $in^3$ | ry $in^3$ | J | CW |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 76 | W27X178 | 178 | 52.5 | 27.8 | 14.1 | 0.725 | 1.19 | 5.92 | 32.9 | 7020 | 570 | 505 | 11.6 | 555 | 122 | 78.8 | 3.25 | 20.1 | 98400 |
| 77 | W27X161 | 161 | 47.6 | 27.6 | 14 | 0.66 | 1.08 | 6.49 | 36.1 | 6310 | 515 | 458 | 11.5 | 497 | 109 | 70.9 | 3.23 | 15.1 | 87300 |
| 78 | W27X146 | 146 | 43.1 | 27.4 | 14 | 0.605 | 0.975 | 7.16 | 39.4 | 5660 | 464 | 414 | 11.5 | 443 | 97.7 | 63.5 | 3.2 | 11.3 | 77200 |
| 79 | W24X192 | 192 | 56.3 | 25.5 | 13 | 0.81 | 1.46 | 4.43 | 26.6 | 6260 | 559 | 491 | 10.5 | 530 | 126 | 81.8 | 3.07 | 30.8 | 76300 |
| 80 | W24X176 | 176 | 51.7 | 25.2 | 12.9 | 0.75 | 1.34 | 4.81 | 28.7 | 5680 | 511 | 450 | 10.5 | 479 | 115 | 74.3 | 3.04 | 23.9 | 68400 |
| 81 | W24X162 | 162 | 47.7 | 25 | 13 | 0.705 | 1.22 | 5.31 | 30.6 | 5170 | 468 | 414 | 10.4 | 443 | 105 | 68.4 | 3.05 | 18.5 | 62600 |
| 82 | W24X146 | 146 | 43 | 24.7 | 12.9 | 0.65 | 1.09 | 5.92 | 33.2 | 4580 | 418 | 371 | 10.3 | 391 | 93.2 | 60.5 | 3.01 | 13.4 | 54600 |
| 83 | W24X131 | 131 | 38.5 | 24.5 | 12.9 | 0.605 | 0.96 | 6.7 | 35.6 | 4020 | 370 | 329 | 10.2 | 340 | 81.5 | 53 | 2.97 | 9.5 | 47100 |
| 84 | W24X117 | 117 | 34.4 | 24.3 | 12.8 | 0.55 | 0.85 | 7.53 | 39.2 | 3540 | 327 | 291 | 10.1 | 297 | 71.4 | 46.5 | 2.94 | 6.72 | 40800 |
| 85 | W24X104 | 104 | 30.6 | 24.1 | 12.8 | 0.5 | 0.75 | 8.5 | 43.1 | 3100 | 289 | 258 | 10.1 | 259 | 62.4 | 40.7 | 2.91 | 4.72 | 35200 |
| 86 | W21X182 | 182 | 53.6 | 22.7 | 12.5 | 0.83 | 1.48 | 4.22 | 22.6 | 4730 | 476 | 417 | 9.4 | 483 | 119 | 77.2 | 3 | 30.7 | 54400 |
| 87 | W21X166 | 166 | 48.8 | 22.5 | 12.4 | 0.75 | 1.36 | 4.57 | 25 | 4280 | 432 | 380 | 9.36 | 435 | 108 | 70 | 2.99 | 23.6 | 48500 |
| 88 | W21X147 | 147 | 43.2 | 22.1 | 12.5 | 0.72 | 1.15 | 5.44 | 26.1 | 3630 | 373 | 329 | 9.17 | 376 | 92.6 | 60.1 | 2.95 | 15.4 | 41100 |
| 89 | W21X132 | 132 | 38.8 | 21.8 | 12.4 | 0.65 | 1.04 | 6.01 | 28.9 | 3220 | 333 | 295 | 9.12 | 333 | 82.3 | 53.5 | 2.93 | 11.3 | 36000 |
| 90 | W21X122 | 122 | 35.9 | 21.7 | 12.4 | 0.6 | 0.96 | 6.45 | 31.3 | 2960 | 307 | 273 | 9.09 | 305 | 75.6 | 49.2 | 2.92 | 8.98 | 32700 |
| 91 | W21X111 | 111 | 32.7 | 21.5 | 12.3 | 0.55 | 0.875 | 7.05 | 34.1 | 2670 | 279 | 249 | 9.05 | 274 | 68.2 | 44.5 | 2.9 | 6.83 | 29200 |
| 92 | W21X101 | 101 | 29.8 | 21.4 | 12.3 | 0.5 | 0.8 | 7.68 | 37.5 | 2420 | 253 | 227 | 9.02 | 248 | 61.7 | 40.3 | 2.89 | 5.21 | 26200 |
| 93 | W18x192 | 192 | 56.4 | 20.4 | 11.5 | 0.96 | 1.75 | 3.27 | 16.7 | 3870 | 442 | 380 | 8.28 | 440 | 119 | 76.8 | 2.79 | 44.7 | 38000 |
| 94 | W18X175 | 175 | 51.3 | 20 | 11.4 | 0.89 | 1.59 | 3.58 | 18 | 3450 | 398 | 344 | 8.2 | 391 | 106 | 68.8 | 2.76 | 33.8 | 33300 |
| 95 | W18X158 | 158 | 46.3 | 19.7 | 11.3 | 0.81 | 1.44 | 3.92 | 19.8 | 3060 | 356 | 310 | 8.12 | 347 | 94.8 | 61.4 | 2.74 | 25.2 | 29000 |
| 96 | W18X143 | 143 | 42.1 | 19.5 | 11.2 | 0.73 | 1.32 | 4.25 | 22 | 2750 | 322 | 282 | 8.09 | 311 | 85.4 | 55.5 | 2.72 | 19.2 | 25700 |
| 97 | W18X130 | 130 | 38.2 | 19.3 | 11.2 | 0.67 | 1.2 | 4.65 | 23.9 | 2460 | 290 | 256 | 8.03 | 278 | 76.7 | 49.9 | 2.7 | 14.5 | 22700 |
| 98 | W18X119 | 119 | 35.1 | 19 | 11.3 | 0.655 | 1.06 | 5.31 | 24.5 | 2190 | 262 | 231 | 7.9 | 253 | 69.1 | 44.9 | 2.69 | 10.6 | 20300 |
| 99 | W18X106 | 106 | 31.1 | 18.7 | 11.2 | 0.59 | 0.94 | 5.96 | 27.2 | 1910 | 230 | 204 | 7.84 | 220 | 60.5 | 39.4 | 2.66 | 7.48 | 17400 |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 100 | W18X97 | 97 | 28.5 | 18.6 | 11.1 | 0.535 | 0.87 | 6.41 | 30 | 1750 | 211 | 188 | 7.82 | 201 | 55.3 | 36.1 | 2.65 | 5.86 | 15800 |
| 101 | W18X86 | 86 | 25.3 | 18.4 | 11.1 | 0.48 | 0.77 | 7.2 | 33.4 | 1530 | 186 | 166 | 7.77 | 175 | 48.4 | 31.6 | 2.63 | 4.1 | 13600 |
| 102 | W18X76 | 76 | 22.3 | 18.2 | 11 | 0.425 | 0.68 | 8.11 | 37.8 | 1330 | 163 | 146 | 7.73 | 152 | 42.2 | 27.6 | 2.61 | 2.83 | 11700 |
| 103 | W16X100 | 100 | 29.5 | 17 | 10.4 | 0.585 | 0.985 | 5.29 | 24.3 | 1490 | 198 | 175 | 7.1 | 186 | 54.9 | 35.7 | 2.51 | 7.73 | 11900 |
| 104 | W16X89 | 89 | 26.2 | 16.8 | 10.4 | 0.525 | 0.875 | 5.92 | 27 | 1300 | 175 | 155 | 7.05 | 163 | 48.1 | 31.4 | 2.49 | 5.45 | 10200 |
| 105 | W16X77 | 77 | 22.6 | 16.5 | 10.3 | 0.455 | 0.76 | 6.77 | 31.2 | 1110 | 150 | 134 | 7 | 138 | 41.1 | 26.9 | 2.47 | 3.57 | 8590 |
| 106 | W16X67 | 67 | 19.7 | 16.3 | 10.2 | 0.395 | 0.665 | 7.7 | 35.9 | 954 | 130 | 117 | 6.96 | 119 | 35.5 | 23.2 | 2.46 | 2.39 | 7300 |
| 107 | W14X193 | 193 | 56.8 | 15.5 | 15.7 | 0.89 | 1.44 | 5.45 | 12.8 | 2400 | 355 | 310 | 6.5 | 931 | 180 | 119 | 4.05 | 34.8 | 45900 |
| 108 | W14X176 | 176 | 51.8 | 15.2 | 15.7 | 0.83 | 1.31 | 5.97 | 13.7 | 2140 | 320 | 281 | 6.43 | 838 | 163 | 107 | 4.02 | 26.5 | 40500 |
| 109 | W14X159 | 159 | 46.7 | 15 | 15.6 | 0.745 | 1.19 | 6.54 | 15.3 | 1900 | 287 | 254 | 6.38 | 748 | 146 | 96.2 | 4 | 19.7 | 35600 |
| 110 | W14X145 | 145 | 42.7 | 14.8 | 15.5 | 0.68 | 1.09 | 7.11 | 16.8 | 1710 | 260 | 232 | 6.33 | 677 | 133 | 87.3 | 3.98 | 15.2 | 31700 |
| 111 | W14X132 | 132 | 38.8 | 14.7 | 14.7 | 0.645 | 1.03 | 7.15 | 17.7 | 1530 | 234 | 209 | 6.28 | 548 | 113 | 74.5 | 3.76 | 12.3 | 25500 |
| 112 | W14X120 | 120 | 35.3 | 14.5 | 14.7 | 0.59 | 0.94 | 7.8 | 19.3 | 1380 | 212 | 190 | 6.24 | 495 | 102 | 67.5 | 3.74 | 9.37 | 22700 |
| 113 | W14X109 | 109 | 32 | 14.3 | 14.6 | 0.525 | 0.86 | 8.49 | 21.7 | 1240 | 192 | 173 | 6.22 | 447 | 92.7 | 61.2 | 3.73 | 7.12 | 20200 |
| 114 | W14X99 | 99 | 29.1 | 14.2 | 14.6 | 0.485 | 0.78 | 9.34 | 23.5 | 1110 | 173 | 157 | 6.17 | 402 | 83.6 | 55.2 | 3.71 | 5.37 | 18000 |
| 115 | W14X90 | 90 | 26.5 | 14 | 14.5 | 0.44 | 0.71 | 10.2 | 25.9 | 999 | 157 | 143 | 6.14 | 362 | 75.6 | 49.9 | 3.7 | 4.06 | 16000 |
| 116 | W14X82 | 82 | 24 | 14.3 | 10.1 | 0.51 | 0.855 | 5.92 | 22.4 | 881 | 139 | 123 | 6.05 | 148 | 44.8 | 29.3 | 2.48 | 5.07 | 6710 |
| 117 | W14X74 | 74 | 21.8 | 14.2 | 10.1 | 0.45 | 0.785 | 6.41 | 25.4 | 795 | 126 | 112 | 6.04 | 134 | 40.5 | 26.6 | 2.48 | 3.87 | 5990 |
| 118 | W14X68 | 68 | 20 | 14 | 10 | 0.415 | 0.72 | 6.97 | 27.5 | 722 | 115 | 103 | 6.01 | 121 | 36.9 | 24.2 | 2.46 | 3.01 | 5380 |
| 119 | W14X61 | 61 | 17.9 | 13.9 | 10 | 0.375 | 0.645 | 7.75 | 30.4 | 640 | 102 | 92.1 | 5.98 | 107 | 32.8 | 21.5 | 2.45 | 2.19 | 4710 |
| 120 | W14X53 | 53 | 15.6 | 13.9 | 8.06 | 0.37 | 0.66 | 6.11 | 30.9 | 541 | 87.1 | 77.8 | 5.89 | 57.7 | 22 | 14.3 | 1.92 | 1.94 | 2540 |
| 121 | W14X48 | 48 | 14.1 | 13.8 | 8.03 | 0.34 | 0.595 | 6.75 | 33.6 | 484 | 78.4 | 70.2 | 5.85 | 51.4 | 19.6 | 12.8 | 1.91 | 1.45 | 2240 |
| 122 | W14X43 | 43 | 12.6 | 13.7 | 8 | 0.305 | 0.53 | 7.54 | 37.4 | 428 | 69.6 | 62.6 | 5.82 | 45.2 | 17.3 | 11.3 | 1.89 | 1.05 | 1950 |
| 123 | W12X190 | 190 | 55.8 | 14.4 | 12.7 | 1.06 | 1.74 | 3.65 | 9.16 | 1890 | 311 | 263 | 5.82 | 589 | 143 | 93 | 3.25 | 48.8 | 23600 |
| 124 | W12X170 | 170 | 50 | 14 | 12.6 | 0.96 | 1.56 | 4.03 | 10.1 | 1650 | 275 | 235 | 5.74 | 517 | 126 | 82.3 | 3.22 | 35.6 | 20100 |
| 125 | W12X152 | 152 | 44.7 | 13.7 | 12.5 | 0.87 | 1.4 | 4.46 | 11.2 | 1430 | 243 | 209 | 5.66 | 454 | 111 | 72.8 | 3.19 | 25.8 | 17200 |
| 126 | W12X136 | 136 | 39.9 | 13.4 | 12.4 | 0.79 | 1.25 | 4.96 | 12.3 | 1240 | 214 | 186 | 5.58 | 398 | 98 | 64.2 | 3.16 | 18.5 | 14700 |
| 127 | W12X120 | 120 | 35.3 | 13.1 | 12.3 | 0.71 | 1.11 | 5.57 | 13.7 | 1070 | 186 | 163 | 5.51 | 345 | 85.4 | 56 | 3.13 | 12.9 | 12400 |
| 128 | W12X106 | 106 | 31.2 | 12.9 | 12.2 | 0.61 | 0.99 | 6.17 | 15.9 | 933 | 164 | 145 | 5.47 | 301 | 75.1 | 49.3 | 3.11 | 9.13 | 10700 |
| 129 | W12X96 | 96 | 28.2 | 12.7 | 12.2 | 0.55 | 0.9 | 6.76 | 17.7 | 833 | 147 | 131 | 5.44 | 270 | 67.5 | 44.4 | 3.09 | 6.85 | 9410 |
| 130 | W12X87 | 87 | 25.6 | 12.5 | 12.1 | 0.515 | 0.81 | 7.48 | 18.9 | 740 | 132 | 118 | 5.38 | 241 | 60.4 | 39.7 | 3.07 | 5.1 | 8270 |
| 131 | W12X79 | 79 | 23.2 | 12.4 | 12.1 | 0.47 | 0.735 | 8.22 | 20.7 | 662 | 119 | 107 | 5.34 | 216 | 54.3 | 35.8 | 3.05 | 3.84 | 7330 |
| 132 | W12X72 | 72 | 21.1 | 12.3 | 12 | 0.43 | 0.67 | 8.99 | 22.6 | 597 | 108 | 97.4 | 5.31 | 195 | 49.2 | 32.4 | 3.04 | 2.93 | 6540 |
| 133 | W12X65 | 65 | 19.1 | 12.1 | 12 | 0.39 | 0.605 | 9.92 | 24.9 | 533 | 96.8 | 87.9 | 5.28 | 174 | 44.1 | 29.1 | 3.02 | 2.18 | 5780 |
| 134 | W12X58 | 58 | 17 | 12.2 | 10 | 0.36 | 0.64 | 7.82 | 27 | 475 | 86.4 | 78 | 5.28 | 107 | 32.5 | 21.4 | 2.51 | 2.1 | 3570 |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 135 | W12X53 | 53 | 15.6 | 12.1 | 10 | 0.345 | 0.575 | 8.69 | 28.1 | 425 | 77.9 | 70.6 | 5.23 | 95.8 | 29.1 | 19.2 | 2.48 | 1.58 | 3160 |
| 136 | W12X50 | 50 | 14.6 | 12.2 | 8.08 | 0.37 | 0.64 | 6.31 | 26.8 | 391 | 71.9 | 64.2 | 5.18 | 56.3 | 21.3 | 13.9 | 1.96 | 1.71 | 1880 |
| 137 | W12X45 | 45 | 13.1 | 12.1 | 8.05 | 0.335 | 0.575 | 7 | 29.6 | 348 | 64.2 | 57.7 | 5.15 | 50 | 19 | 12.4 | 1.95 | 1.26 | 1650 |
| 138 | W12X40 | 40 | 11.7 | 11.9 | 8.01 | 0.295 | 0.515 | 7.77 | 33.6 | 307 | 57 | 51.5 | 5.13 | 44.1 | 16.8 | 11 | 1.94 | 0.906 | 1440 |
| 139 | W12X35 | 35 | 10.3 | 12.5 | 6.56 | 0.3 | 0.52 | 6.31 | 36.2 | 285 | 51.2 | 45.6 | 5.25 | 24.5 | 11.5 | 7.47 | 1.54 | 0.741 | 879 |
| 140 | W12X30 | 30 | 8.79 | 12.3 | 6.52 | 0.26 | 0.44 | 7.41 | 41.8 | 238 | 43.1 | 38.6 | 5.21 | 20.3 | 9.56 | 6.24 | 1.52 | 0.457 | 720 |
| 141 | W12X26 | 26 | 7.65 | 12.2 | 6.49 | 0.23 | 0.38 | 8.54 | 47.2 | 204 | 37.2 | 33.4 | 5.17 | 17.3 | 8.17 | 5.34 | 1.51 | 0.3 | 607 |
| 142 | W10X112 | 112 | 32.9 | 11.4 | 10.4 | 0.755 | 1.25 | 4.17 | 10.4 | 716 | 147 | 126 | 4.66 | 236 | 69.2 | 45.3 | 2.68 | 15.1 | 6020 |
| 143 | W10X100 | 100 | 29.4 | 11.1 | 10.3 | 0.68 | 1.12 | 4.62 | 11.6 | 623 | 130 | 112 | 4.6 | 207 | 61 | 40 | 2.65 | 10.9 | 5150 |
| 144 | W10X88 | 88 | 25.9 | 10.8 | 10.3 | 0.605 | 0.99 | 5.18 | 13 | 534 | 113 | 98.5 | 4.54 | 179 | 53.1 | 34.8 | 2.63 | 7.53 | 4330 |
| 145 | W10X77 | 77 | 22.6 | 10.6 | 10.2 | 0.53 | 0.87 | 5.86 | 14.8 | 455 | 97.6 | 85.9 | 4.49 | 154 | 45.9 | 30.1 | 2.6 | 5.11 | 3630 |
| 146 | W10X68 | 68 | 20 | 10.4 | 10.1 | 0.47 | 0.77 | 6.58 | 16.7 | 394 | 85.3 | 75.7 | 4.44 | 134 | 40.1 | 26.4 | 2.59 | 3.56 | 3100 |
| 147 | W10X60 | 60 | 17.6 | 10.2 | 10.1 | 0.42 | 0.68 | 7.41 | 18.7 | 341 | 74.6 | 66.7 | 4.39 | 116 | 35 | 23 | 2.57 | 2.48 | 2640 |
| 148 | W10X54 | 54 | 15.8 | 10.1 | 10 | 0.37 | 0.615 | 8.15 | 21.2 | 303 | 66.6 | 60 | 4.37 | 103 | 31.3 | 20.6 | 2.56 | 1.82 | 2320 |
| 149 | W10X49 | 49 | 14.4 | 10 | 10 | 0.34 | 0.56 | 8.93 | 23.1 | 272 | 60.4 | 54.6 | 4.35 | 93.4 | 28.3 | 18.7 | 2.54 | 1.39 | 2070 |
| 150 | W10X45 | 45 | 13.3 | 10.1 | 8.02 | 0.35 | 0.62 | 6.47 | 22.5 | 248 | 54.9 | 49.1 | 4.32 | 53.4 | 20.3 | 13.3 | 2.01 | 1.51 | 1200 |
| 151 | W10X39 | 39 | 11.5 | 9.92 | 7.99 | 0.315 | 0.53 | 7.53 | 25 | 209 | 46.8 | 42.1 | 4.27 | 45 | 17.2 | 11.3 | 1.98 | 0.976 | 992 |
| 152 | W10X33 | 33 | 9.71 | 9.73 | 7.96 | 0.29 | 0.435 | 9.15 | 27.1 | 171 | 38.8 | 35 | 4.19 | 36.6 | 14 | 9.2 | 1.94 | 0.583 | 791 |
| 153 | W10X30 | 30 | 8.84 | 10.5 | 5.81 | 0.3 | 0.51 | 5.7 | 29.5 | 170 | 36.6 | 32.4 | 4.38 | 16.7 | 8.84 | 5.75 | 1.37 | 0.622 | 414 |
| 154 | W10X26 | 26 | 7.61 | 10.3 | 5.77 | 0.26 | 0.44 | 6.56 | 34 | 144 | 31.3 | 27.9 | 4.35 | 14.1 | 7.5 | 4.89 | 1.36 | 0.402 | 345 |
| 155 | W10X22 | 22 | 6.49 | 10.2 | 5.75 | 0.24 | 0.36 | 7.99 | 36.9 | 118 | 26 | 23.2 | 4.27 | 11.4 | 6.1 | 3.97 | 1.33 | 0.239 | 275 |
| 156 | W8X67 | 67 | 19.7 | 9 | 8.28 | 0.57 | 0.935 | 4.43 | 11.1 | 272 | 70.1 | 60.4 | 3.72 | 88.6 | 32.7 | 21.4 | 2.12 | 5.05 | 1440 |
| 157 | W8X58 | 58 | 17.1 | 8.75 | 8.22 | 0.51 | 0.81 | 5.07 | 12.4 | 228 | 59.8 | 52 | 3.65 | 75.1 | 27.9 | 18.3 | 2.1 | 3.33 | 1180 |
| 158 | W8X48 | 48 | 14.1 | 8.5 | 8.11 | 0.4 | 0.685 | 5.92 | 15.9 | 184 | 49 | 43.2 | 3.61 | 60.9 | 22.9 | 15 | 2.08 | 1.96 | 931 |
| 159 | W8X40 | 40 | 11.7 | 8.25 | 8.07 | 0.36 | 0.56 | 7.21 | 17.6 | 146 | 39.8 | 35.5 | 3.53 | 49.1 | 18.5 | 12.2 | 2.04 | 1.12 | 726 |
| 160 | W8X35 | 35 | 10.3 | 8.12 | 8.02 | 0.31 | 0.495 | 8.1 | 20.5 | 127 | 34.7 | 31.2 | 3.51 | 42.6 | 16.1 | 10.6 | 2.03 | 0.769 | 619 |
| 161 | W8X31 | 31 | 9.12 | 8 | 8 | 0.285 | 0.435 | 9.19 | 22.3 | 110 | 30.4 | 27.5 | 3.47 | 37.1 | 14.1 | 9.27 | 2.02 | 0.536 | 530 |
| 162 | W8X28 | 28 | 8.24 | 8.06 | 6.54 | 0.285 | 0.465 | 7.03 | 22.3 | 98 | 27.2 | 24.3 | 3.45 | 21.7 | 10.1 | 6.63 | 1.62 | 0.537 | 312 |
| 163 | W8X24 | 24 | 7.08 | 7.93 | 6.5 | 0.245 | 0.4 | 8.12 | 25.9 | 82.7 | 23.1 | 20.9 | 3.42 | 18.3 | 8.57 | 5.63 | 1.61 | 0.346 | 259 |
| 164 | W8X21 | 21 | 6.16 | 8.28 | 5.27 | 0.25 | 0.4 | 6.59 | 27.5 | 75.3 | 20.4 | 18.2 | 3.49 | 9.77 | 5.69 | 3.71 | 1.26 | 0.282 | 152 |
| 165 | W8X18 | 18 | 5.26 | 8.14 | 5.25 | 0.23 | 0.33 | 7.95 | 29.9 | 61.9 | 17 | 15.2 | 3.43 | 7.97 | 4.66 | 3.04 | 1.23 | 0.172 | 122 |
| 166 | W8X15 | 15 | 4.44 | 8.11 | 4.01 | 0.245 | 0.315 | 6.37 | 28.1 | 48 | 13.6 | 11.8 | 3.29 | 3.41 | 2.67 | 1.7 | 0.876 | 0.137 | 51.8 |
| 167 | W8X13 | 13 | 3.84 | 7.99 | 4 | 0.23 | 0.255 | 7.84 | 29.9 | 39.6 | 11.4 | 9.91 | 3.21 | 2.73 | 2.15 | 1.37 | 0.843 | 0.0871 | 40.8 |
| 168 | W8X10 | 10 | 2.96 | 7.89 | 3.94 | 0.17 | 0.205 | 9.61 | 40.5 | 30.8 | 8.87 | 7.81 | 3.22 | 2.09 | 1.66 | 1.06 | 0.841 | 0.0426 | 30.9 |

## 2.2 Beam catalog sections:

| No. | AISC Sections < 200 lb | Weigth (W) lb/ft | Area (A) $in^2$ | Depth (D) in | Width (BF) in | $t_w$ in | $t_f$ in | $b_f / 2t_f$ | $h / t_w$ | Ix $in^4$ | Zx $in^3$ | Sx $in^3$ | rx $in^3$ | Iy $in^4$ | Zy $in^3$ | Sy $in^3$ | ry $in^3$ | J | CW |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | W40X199 | 199 | 58.5 | 38.7 | 15.8 | 0.650 | 1.07 | 7.39 | 52.6 | 14900 | 869 | 770 | 16.0 | 695 | 137 | 88.2 | 3.45 | 18.3 | 246000 |
| 2 | W40X183 | 183 | 53.3 | 39.0 | 11.8 | 0.650 | 1.20 | 4.92 | 52.6 | 13200 | 774 | 675 | 15.7 | 331 | 88.3 | 56.0 | 2.49 | 19.3 | 118000 |
| 3 | W40X167 | 167 | 49.2 | 38.6 | 11.8 | 0.650 | 1.03 | 5.76 | 52.6 | 11600 | 693 | 600 | 15.3 | 283 | 76.0 | 47.9 | 2.40 | 14.0 | 99700 |
| 4 | W40X149 | 149 | 43.8 | 38.2 | 11.8 | 0.630 | 0.830 | 7.11 | 54.3 | 9800 | 598 | 513 | 15.0 | 229 | 62.2 | 38.8 | 2.29 | 9.36 | 80000 |
| 5 | W36X194 | 194 | 57.0 | 36.5 | 12.1 | 0.765 | 1.26 | 4.81 | 42.4 | 12100 | 767 | 664 | 14.6 | 375 | 97.7 | 61.9 | 2.56 | 22.2 | 116000 |
| 6 | W36X182 | 182 | 53.6 | 36.3 | 12.1 | 0.725 | 1.18 | 5.12 | 44.8 | 11300 | 718 | 623 | 14.5 | 347 | 90.7 | 57.6 | 2.55 | 18.5 | 107000 |
| 7 | W36X170 | 170 | 50.1 | 36.2 | 12.0 | 0.680 | 1.10 | 5.47 | 47.7 | 10500 | 668 | 581 | 14.5 | 320 | 83.8 | 53.2 | 2.53 | 15.1 | 98500 |
| 8 | W36X160 | 160 | 47.0 | 36.0 | 12.0 | 0.650 | 1.02 | 5.88 | 49.9 | 9760 | 624 | 542 | 14.4 | 295 | 77.3 | 49.1 | 2.50 | 12.4 | 90200 |
| 9 | W36X150 | 150 | 44.2 | 35.9 | 12.0 | 0.625 | 0.940 | 6.37 | 51.9 | 9040 | 581 | 504 | 14.3 | 270 | 70.9 | 45.1 | 2.47 | 10.1 | 82200 |
| 10 | W36X135 | 135 | 39.7 | 35.6 | 12.0 | 0.600 | 0.790 | 7.56 | 54.1 | 7800 | 509 | 439 | 14.0 | 225 | 59.7 | 37.7 | 2.38 | 7.00 | 68100 |
| 11 | W33X169 | 169 | 49.5 | 33.8 | 11.5 | 0.670 | 1.22 | 4.71 | 44.7 | 9290 | 629 | 549 | 13.7 | 310 | 84.4 | 53.9 | 2.50 | 17.7 | 82400 |
| 12 | W33X152 | 152 | 44.8 | 33.5 | 11.6 | 0.635 | 1.06 | 5.48 | 47.2 | 8160 | 559 | 487 | 13.5 | 273 | 73.9 | 47.2 | 2.47 | 12.4 | 71700 |
| 13 | W33X141 | 141 | 41.6 | 33.3 | 11.5 | 0.605 | 0.960 | 6.01 | 49.6 | 7450 | 514 | 448 | 13.4 | 246 | 66.9 | 42.7 | 2.43 | 9.70 | 64400 |
| 14 | W33X130 | 130 | 38.3 | 33.1 | 11.5 | 0.580 | 0.855 | 6.73 | 51.7 | 6710 | 467 | 406 | 13.2 | 218 | 59.5 | 37.9 | 2.39 | 7.37 | 56600 |
| 15 | W33X118 | 118 | 34.7 | 32.9 | 11.5 | 0.550 | 0.740 | 7.76 | 54.5 | 5900 | 415 | 359 | 13.0 | 187 | 51.3 | 32.6 | 2.32 | 5.30 | 48300 |
| 16 | W30X191 | 191 | 56.3 | 30.7 | 15.0 | 0.710 | 1.19 | 6.35 | 37.7 | 9200 | 675 | 600 | 12.8 | 673 | 138 | 89.5 | 3.46 | 21.0 | 146000 |
| 17 | W30X173 | 173 | 51.0 | 30.4 | 15.0 | 0.655 | 1.07 | 7.04 | 40.8 | 8230 | 607 | 541 | 12.7 | 598 | 123 | 79.8 | 3.42 | 15.6 | 129000 |
| 18 | W30X148 | 148 | 43.5 | 30.7 | 10.5 | 0.650 | 1.18 | 4.44 | 41.6 | 6680 | 500 | 436 | 12.4 | 227 | 68.0 | 43.3 | 2.28 | 14.5 | 49400 |
| 19 | W30X132 | 132 | 38.9 | 30.3 | 10.5 | 0.615 | 1.00 | 5.27 | 43.9 | 5770 | 437 | 380 | 12.2 | 196 | 58.4 | 37.2 | 2.25 | 9.72 | 42100 |
| 20 | W30X124 | 124 | 36.5 | 30.2 | 10.5 | 0.585 | 0.930 | 5.65 | 46.2 | 5360 | 408 | 355 | 12.1 | 181 | 54.0 | 34.4 | 2.23 | 7.99 | 38600 |
| 21 | W30X116 | 116 | 34.2 | 30.0 | 10.5 | 0.565 | 0.850 | 6.17 | 47.8 | 4930 | 378 | 329 | 12.0 | 164 | 49.2 | 31.3 | 2.19 | 6.43 | 34900 |
| 22 | W30X108 | 108 | 31.7 | 29.8 | 10.5 | 0.545 | 0.760 | 6.89 | 49.6 | 4470 | 346 | 299 | 11.9 | 146 | 43.9 | 27.9 | 2.15 | 4.99 | 30900 |
| 23 | W30X99 | 99.0 | 29.1 | 29.7 | 10.5 | 0.520 | 0.670 | 7.80 | 51.9 | 3990 | 312 | 269 | 11.7 | 128 | 38.6 | 24.5 | 2.10 | 3.77 | 26800 |
| 24 | W30X90 | 90.0 | 26.4 | 29.5 | 10.4 | 0.470 | 0.610 | 8.52 | 57.5 | 3610 | 283 | 245 | 11.7 | 115 | 34.7 | 22.1 | 2.09 | 2.84 | 24000 |
| 25 | W27X194 | 194 | 57.2 | 28.1 | 14.0 | 0.750 | 1.34 | 5.24 | 31.8 | 7860 | 631 | 559 | 11.7 | 619 | 136 | 88.1 | 3.29 | 27.1 | 111000 |
| 26 | W27X129 | 129 | 37.8 | 27.6 | 10.0 | 0.610 | 1.10 | 4.55 | 39.7 | 4760 | 395 | 345 | 11.2 | 184 | 57.6 | 36.8 | 2.21 | 11.1 | 32500 |
| 27 | W27X114 | 114 | 33.5 | 27.3 | 10.1 | 0.570 | 0.930 | 5.41 | 42.5 | 4080 | 343 | 299 | 11.0 | 159 | 49.3 | 31.5 | 2.18 | 7.33 | 27600 |
| 28 | W27X102 | 102 | 30.0 | 27.1 | 10.0 | 0.515 | 0.830 | 6.03 | 47.1 | 3620 | 305 | 267 | 11.0 | 139 | 43.4 | 27.8 | 2.15 | 5.28 | 24000 |
| 29 | W27X94 | 94.0 | 27.7 | 26.9 | 10.0 | 0.490 | 0.745 | 6.70 | 49.5 | 3270 | 278 | 243 | 10.9 | 124 | 38.8 | 24.8 | 2.12 | 4.03 | 21300 |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 30 | W27X84 | 84.0 | 24.8 | 26.7 | 10.0 | 0.460 | 0.640 | 7.78 | 52.7 | 2850 | 244 | 213 | 10.7 | 106 | 33.2 | 21.2 | 2.07 | 2.81 | 17900 |
| 31 | W24X103 | 103 | 30.3 | 24.5 | 9.00 | 0.550 | 0.980 | 4.59 | 39.2 | 3000 | 280 | 245 | 10.0 | 119 | 41.5 | 26.5 | 1.99 | 7.07 | 16600 |
| 32 | W24X94 | 94.0 | 27.7 | 24.3 | 9.07 | 0.515 | 0.875 | 5.18 | 41.9 | 2700 | 254 | 222 | 9.87 | 109 | 37.5 | 24.0 | 1.98 | 5.26 | 15000 |
| 33 | W24X84 | 84.0 | 24.7 | 24.1 | 9.02 | 0.470 | 0.770 | 5.86 | 45.9 | 2370 | 224 | 196 | 9.79 | 94.4 | 32.6 | 20.9 | 1.95 | 3.70 | 12800 |
| 34 | W24X76 | 76.0 | 22.4 | 23.9 | 8.99 | 0.440 | 0.680 | 6.61 | 49.0 | 2100 | 200 | 176 | 9.69 | 82.5 | 28.6 | 18.4 | 1.92 | 2.68 | 11100 |
| 35 | W24X68 | 68.0 | 20.1 | 23.7 | 8.97 | 0.415 | 0.585 | 7.66 | 52.0 | 1830 | 177 | 154 | 9.55 | 70.4 | 24.5 | 15.7 | 1.87 | 1.87 | 9430 |
| 36 | W24X62 | 62.0 | 18.2 | 23.7 | 7.04 | 0.430 | 0.590 | 5.97 | 50.1 | 1550 | 153 | 131 | 9.23 | 34.5 | 15.7 | 9.80 | 1.38 | 1.71 | 4620 |
| 37 | W24X55 | 55.0 | 16.2 | 23.6 | 7.01 | 0.395 | 0.505 | 6.94 | 54.6 | 1350 | 134 | 114 | 9.11 | 29.1 | 13.3 | 8.30 | 1.34 | 1.18 | 3870 |
| 38 | W21X93 | 93.0 | 27.3 | 21.6 | 8.42 | 0.580 | 0.930 | 4.53 | 32.3 | 2070 | 221 | 192 | 8.70 | 92.9 | 34.7 | 22.1 | 1.84 | 6.03 | 9940 |
| 39 | W21X83 | 83.0 | 24.3 | 21.4 | 8.36 | 0.515 | 0.835 | 5.00 | 36.4 | 1830 | 196 | 171 | 8.67 | 81.4 | 30.5 | 19.5 | 1.83 | 4.34 | 8630 |
| 40 | W21X73 | 73.0 | 21.5 | 21.2 | 8.30 | 0.455 | 0.740 | 5.60 | 41.2 | 1600 | 172 | 151 | 8.64 | 70.6 | 26.6 | 17.0 | 1.81 | 3.02 | 7410 |
| 41 | W21X68 | 68.0 | 20.0 | 21.1 | 8.27 | 0.430 | 0.685 | 6.04 | 43.6 | 1480 | 160 | 140 | 8.60 | 64.7 | 24.4 | 15.7 | 1.80 | 2.45 | 6760 |
| 42 | W21X62 | 62.0 | 18.3 | 21.0 | 8.24 | 0.400 | 0.615 | 6.70 | 46.9 | 1330 | 144 | 127 | 8.54 | 57.5 | 21.7 | 14.0 | 1.77 | 1.83 | 5960 |
| 43 | W21X55 | 55.0 | 16.2 | 20.8 | 8.22 | 0.375 | 0.522 | 7.87 | 50.0 | 1140 | 126 | 110 | 8.40 | 48.4 | 18.4 | 11.8 | 1.73 | 1.24 | 4980 |
| 44 | W21X48 | 48.0 | 14.1 | 20.6 | 8.14 | 0.350 | 0.430 | 9.47 | 53.6 | 959 | 107 | 93.0 | 8.24 | 38.7 | 14.9 | 9.52 | 1.66 | 0.803 | 3950 |
| 45 | W21X57 | 57.0 | 16.7 | 21.1 | 6.56 | 0.405 | 0.650 | 5.04 | 46.3 | 1170 | 129 | 111 | 8.36 | 30.6 | 14.8 | 9.35 | 1.35 | 1.77 | 3190 |
| 46 | W21X50 | 50.0 | 14.7 | 20.8 | 6.53 | 0.380 | 0.535 | 6.10 | 49.4 | 984 | 110 | 94.5 | 8.18 | 24.9 | 12.2 | 7.64 | 1.30 | 1.14 | 2570 |
| 47 | W21X44 | 44.0 | 13.0 | 20.7 | 6.50 | 0.350 | 0.450 | 7.22 | 53.6 | 843 | 95.4 | 81.6 | 8.06 | 20.7 | 10.2 | 6.37 | 1.26 | 0.770 | 2110 |
| 48 | W18X71 | 71.0 | 20.8 | 18.5 | 7.64 | 0.495 | 0.810 | 4.71 | 32.4 | 1170 | 146 | 127 | 7.50 | 60.3 | 24.7 | 15.8 | 1.70 | 3.49 | 4700 |
| 49 | W18X65 | 65.0 | 19.1 | 18.4 | 7.59 | 0.450 | 0.750 | 5.06 | 35.7 | 1070 | 133 | 117 | 7.49 | 54.8 | 22.5 | 14.4 | 1.69 | 2.73 | 4240 |
| 50 | W18X60 | 60.0 | 17.6 | 18.2 | 7.56 | 0.415 | 0.695 | 5.44 | 38.7 | 984 | 123 | 108 | 7.47 | 50.1 | 20.6 | 13.3 | 1.68 | 2.17 | 3850 |
| 51 | W18X55 | 55.0 | 16.2 | 18.1 | 7.53 | 0.390 | 0.630 | 5.98 | 41.1 | 890 | 112 | 98.3 | 7.41 | 44.9 | 18.5 | 11.9 | 1.67 | 1.66 | 3430 |
| 52 | W18X50 | 50.0 | 14.7 | 18.0 | 7.50 | 0.355 | 0.570 | 6.57 | 45.2 | 800 | 101 | 88.9 | 7.38 | 40.1 | 16.6 | 10.7 | 1.65 | 1.24 | 3040 |
| 53 | W18X46 | 46.0 | 13.5 | 18.1 | 6.06 | 0.360 | 0.605 | 5.01 | 44.6 | 712 | 90.7 | 78.8 | 7.25 | 22.5 | 11.7 | 7.43 | 1.29 | 1.22 | 1720 |
| 54 | W18X40 | 40.0 | 11.8 | 17.9 | 6.02 | 0.315 | 0.525 | 5.73 | 50.9 | 612 | 78.4 | 68.4 | 7.21 | 19.1 | 10.0 | 6.35 | 1.27 | 0.810 | 1440 |
| 55 | W18X35 | 35.0 | 10.3 | 17.7 | 6.00 | 0.300 | 0.425 | 7.06 | 53.5 | 510 | 66.5 | 57.6 | 7.04 | 15.3 | 8.06 | 5.12 | 1.22 | 0.506 | 1140 |
| 56 | W16X57 | 57.0 | 16.8 | 16.4 | 7.12 | 0.430 | 0.715 | 4.98 | 33.0 | 758 | 105 | 92.2 | 6.72 | 43.1 | 18.9 | 12.1 | 1.60 | 2.22 | 2660 |
| 57 | W16X50 | 50.0 | 14.7 | 16.3 | 7.07 | 0.380 | 0.630 | 5.61 | 37.4 | 659 | 92.0 | 81.0 | 6.68 | 37.2 | 16.3 | 10.5 | 1.59 | 1.52 | 2270 |
| 58 | W16X45 | 45.0 | 13.3 | 16.1 | 7.04 | 0.345 | 0.565 | 6.23 | 41.1 | 586 | 82.3 | 72.7 | 6.65 | 32.8 | 14.5 | 9.34 | 1.57 | 1.11 | 1990 |
| 59 | W16X40 | 40.0 | 11.8 | 16.0 | 7.00 | 0.305 | 0.505 | 6.93 | 46.5 | 518 | 73.0 | 64.7 | 6.63 | 28.9 | 12.7 | 8.25 | 1.57 | 0.794 | 1730 |
| 60 | W16X36 | 36.0 | 10.6 | 15.9 | 6.99 | 0.295 | 0.430 | 8.12 | 48.1 | 448 | 64.0 | 56.5 | 6.51 | 24.5 | 10.8 | 7.00 | 1.52 | 0.545 | 1460 |
| 61 | W16X31 | 31.0 | 9.13 | 15.9 | 5.53 | 0.275 | 0.440 | 6.28 | 51.6 | 375 | 54.0 | 47.2 | 6.41 | 12.4 | 7.03 | 4.49 | 1.17 | 0.461 | 739 |
| 62 | W16X26 | 26.0 | 7.68 | 15.7 | 5.50 | 0.250 | 0.345 | 7.97 | 56.8 | 301 | 44.2 | 38.4 | 6.26 | 9.59 | 5.48 | 3.49 | 1.12 | 0.262 | 565 |
| 63 | W14X38 | 38.0 | 11.2 | 14.1 | 6.77 | 0.310 | 0.515 | 6.57 | 39.6 | 385 | 61.5 | 54.6 | 5.87 | 26.7 | 12.1 | 7.88 | 1.55 | 0.798 | 1230 |
| 64 | W14X34 | 34.0 | 10.0 | 14.0 | 6.75 | 0.285 | 0.455 | 7.41 | 43.1 | 340 | 54.6 | 48.6 | 5.83 | 23.3 | 10.6 | 6.91 | 1.53 | 0.569 | 1070 |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 65 | W14X30 | 30.0 | 8.85 | 13.8 | 6.73 | 0.270 | 0.385 | 8.74 | 45.4 | 291 | 47.3 | 42.0 | 5.73 | 19.6 | 8.99 | 5.82 | 1.49 | 0.380 | 887 |
| 66 | W14X26 | 26.0 | 7.69 | 13.9 | 5.03 | 0.255 | 0.420 | 5.98 | 48.1 | 245 | 40.2 | 35.3 | 5.65 | 8.91 | 5.54 | 3.55 | 1.08 | 0.358 | 405 |
| 67 | W14X22 | 22.0 | 6.49 | 13.7 | 5.00 | 0.230 | 0.335 | 7.46 | 53.3 | 199 | 33.2 | 29.0 | 5.54 | 7.00 | 4.39 | 2.80 | 1.04 | 0.208 | 314 |
| 68 | W12X22 | 22.0 | 6.48 | 12.3 | 4.03 | 0.260 | 0.425 | 4.74 | 41.8 | 156 | 29.3 | 25.4 | 4.91 | 4.66 | 3.66 | 2.31 | 0.848 | 0.293 | 164 |
| 69 | W12X19 | 19.0 | 5.57 | 12.2 | 4.01 | 0.235 | 0.350 | 5.72 | 46.2 | 130 | 24.7 | 21.3 | 4.82 | 3.76 | 2.98 | 1.88 | 0.822 | 0.180 | 131 |
| 70 | W12X16 | 16.0 | 4.71 | 12.0 | 3.99 | 0.220 | 0.265 | 7.53 | 49.4 | 103 | 20.1 | 17.1 | 4.67 | 2.82 | 2.26 | 1.41 | 0.773 | 0.103 | 96.9 |
| 71 | W12X14 | 14.0 | 4.2 | 11.9 | 4.0 | 0.2 | 0.2 | 8.8 | 54.3 | 88.6 | 17.4 | 14.9 | 4.6 | 2.4 | 1.9 | 1.2 | 0.8 | 0.1 | 80.4 |
| 72 | W10X19 | 19.0 | 5.6 | 10.2 | 4.0 | 0.3 | 0.4 | 5.1 | 35.4 | 96.3 | 21.6 | 18.8 | 4.1 | 4.3 | 3.4 | 2.1 | 0.9 | 0.2 | 104.0 |
| 73 | W10X17 | 17.0 | 5.0 | 10.1 | 4.0 | 0.2 | 0.3 | 6.1 | 36.9 | 81.9 | 18.7 | 16.2 | 4.1 | 3.6 | 2.8 | 1.8 | 0.8 | 0.2 | 85.1 |
| 74 | W10X15 | 15.0 | 4.4 | 10.0 | 4.0 | 0.2 | 0.3 | 7.4 | 38.5 | 68.9 | 16.0 | 13.8 | 4.0 | 2.9 | 2.3 | 1.5 | 0.8 | 0.1 | 68.3 |
| 75 | W10X12 | 12.0 | 3.5 | 9.9 | 4.0 | 0.2 | 0.2 | 9.4 | 46.6 | 53.8 | 12.6 | 10.9 | 3.9 | 2.2 | 1.7 | 1.1 | 0.8 | 0.1 | 50.9 |